Lecture 11:

Directory-Based Coherence + Implementing Synchronization

Parallel Computing
Stanford CS149, Fall 2019

What's Due

- Nov 1
 - Assignment 3: A Simple Renderer in CUDA

- Nov 4
 - Written Assignment 3

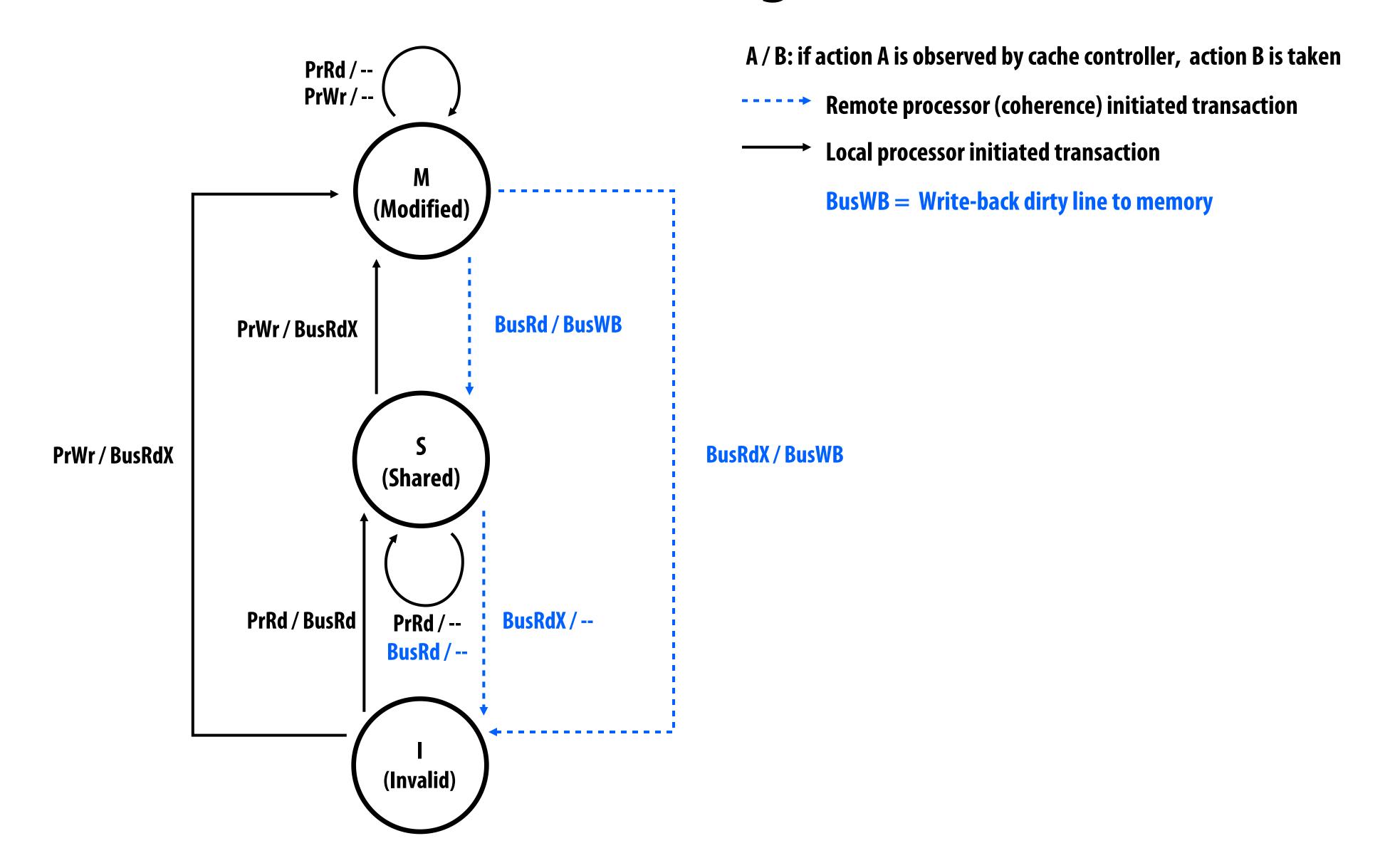
- Nov 5
 - Midterm
 - Open book, open notes
 - Review session on Nov 3

Today's topics

A discussion of directory-based cache coherence

- Efficiently implementing synchronization primitives
 - Primitives for ensuring mutual exclusion
 - Locks
 - Atomic primitives (e.g., atomic_add)
 - Transactions (later in the course)
 - Primitives for event signaling
 - Barriers
- OpenMP
 - Parallelizing loops

Review: MSI state transition diagram *



^{*} Remember, all caches are carrying out this logic independently to maintain coherence

Example

Hit/Miss		Bus	P0 state	P1 state
	PO: LD X			
	PO: LD X			
	P0: ST X ← 1			
	P0: ST X ← 2			
	P1: ST X ← 3			
	P1: LD X			
	PO: LD X			
	P0: ST X ← 4			
	P1: LD X			
	PO: LD Y			
	P0: STY ← 1			
	P1: ST Y ← 2			

Consider this sequence of loads and stores to addresses X and Y by processors P0 and P1

Assume that X and Y contain value 0 at start of execution

Directory-based cache coherence

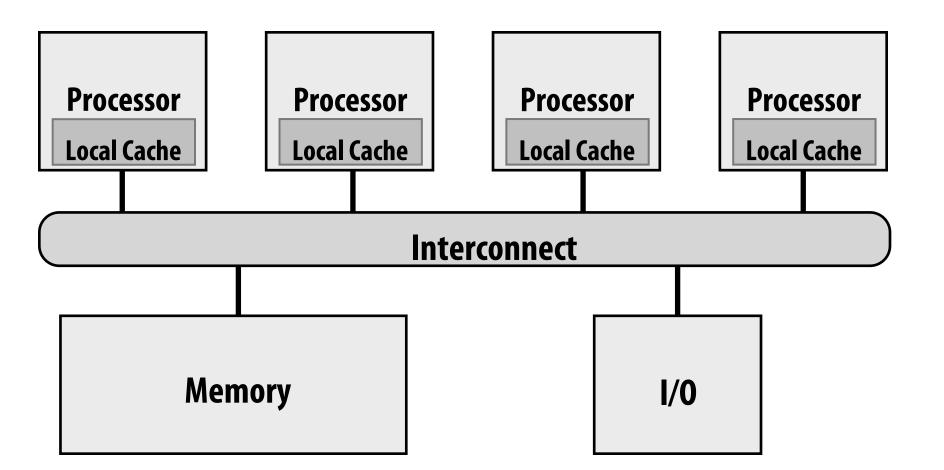
What you should know

- What limits the scalability of snooping-based approaches to cache coherence?
- How does a directory-based scheme avoid these problems?
- How can the storage overhead of the directory structure be reduced? (and at what cost?)

Implementing cache coherence

The snooping cache coherence protocols discussed last week relied on <u>broadcasting</u> coherence information to all processors over the chip interconnect.

Every time a cache miss occurred, the triggering cache communicated with all other caches!

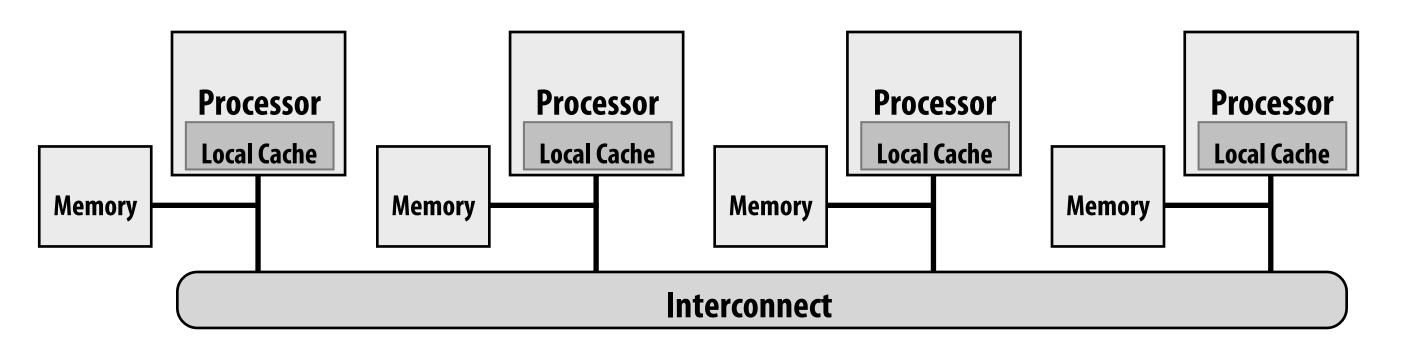


We discussed what information was communicated and what actions were taken to implement the coherence protocol.

We discussed breifly how to implement broadcasts on an interconnect.

- One example is to use a shared bus for the interconnect
- Efficient broadcast
- Scalability of buses is limited by bus bandwidth

Problem: scaling cache coherence to large machines



Recall idea of non-uniform access shared memory systems (NUMA): locating regions of memory near the processors increases scalability: it yields higher aggregate bandwidth and reduced latency (especially when there is locality in the application)

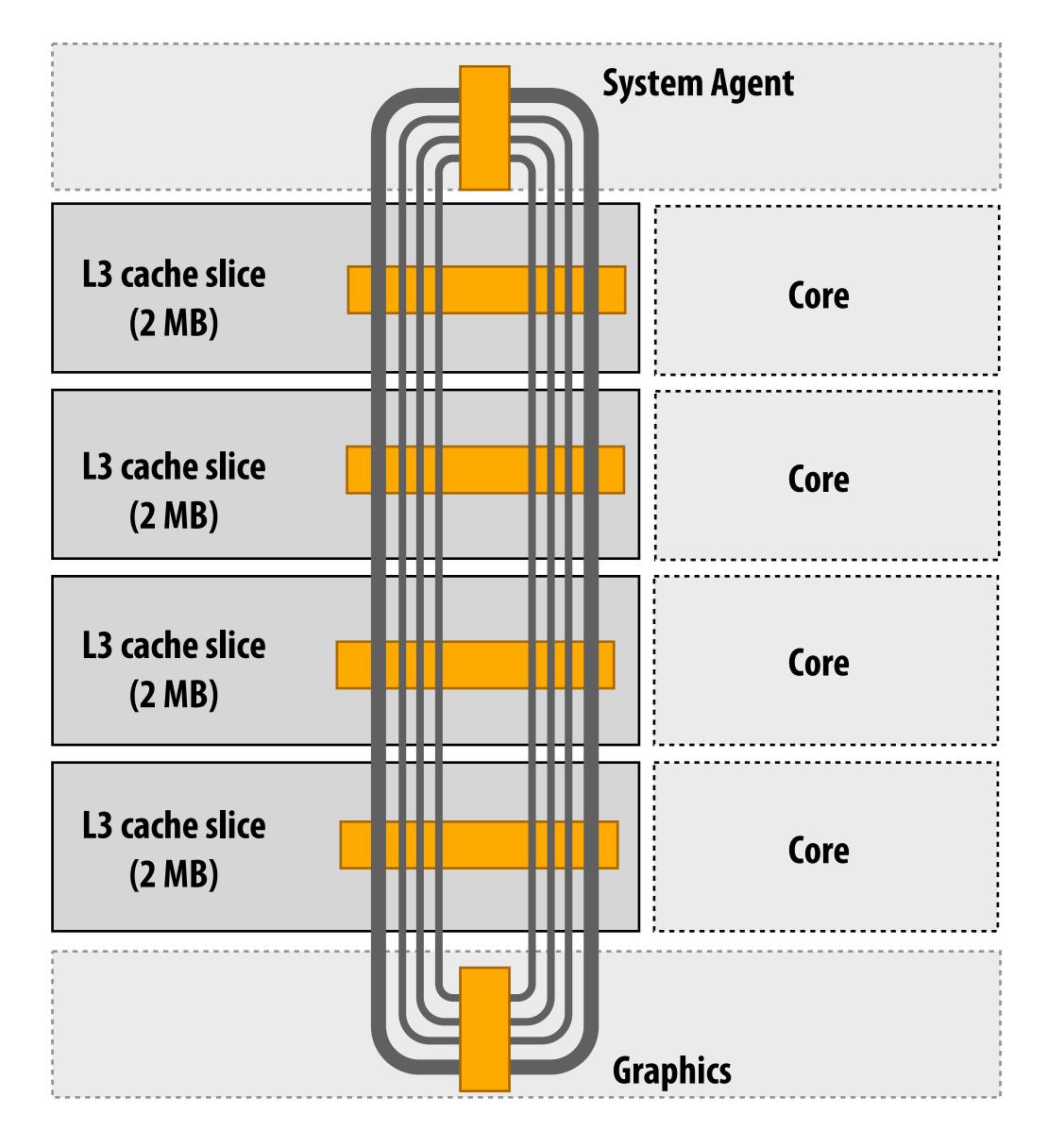
But... efficiency of NUMA system does little good if the coherence protocol can't also be scaled!

Consider this case: processor accesses nearby memory (good...), but to ensure coherence still must broadcast to all other processors it is doing so (bad...)

Some common terminology:

- cc-NUMA = "cache-coherent, non-uniform memory access"
- Distributed shared memory system (DSM): cache coherent, shared address space, but architecture implemented by physically distributed memories

Intel's ring interconnect

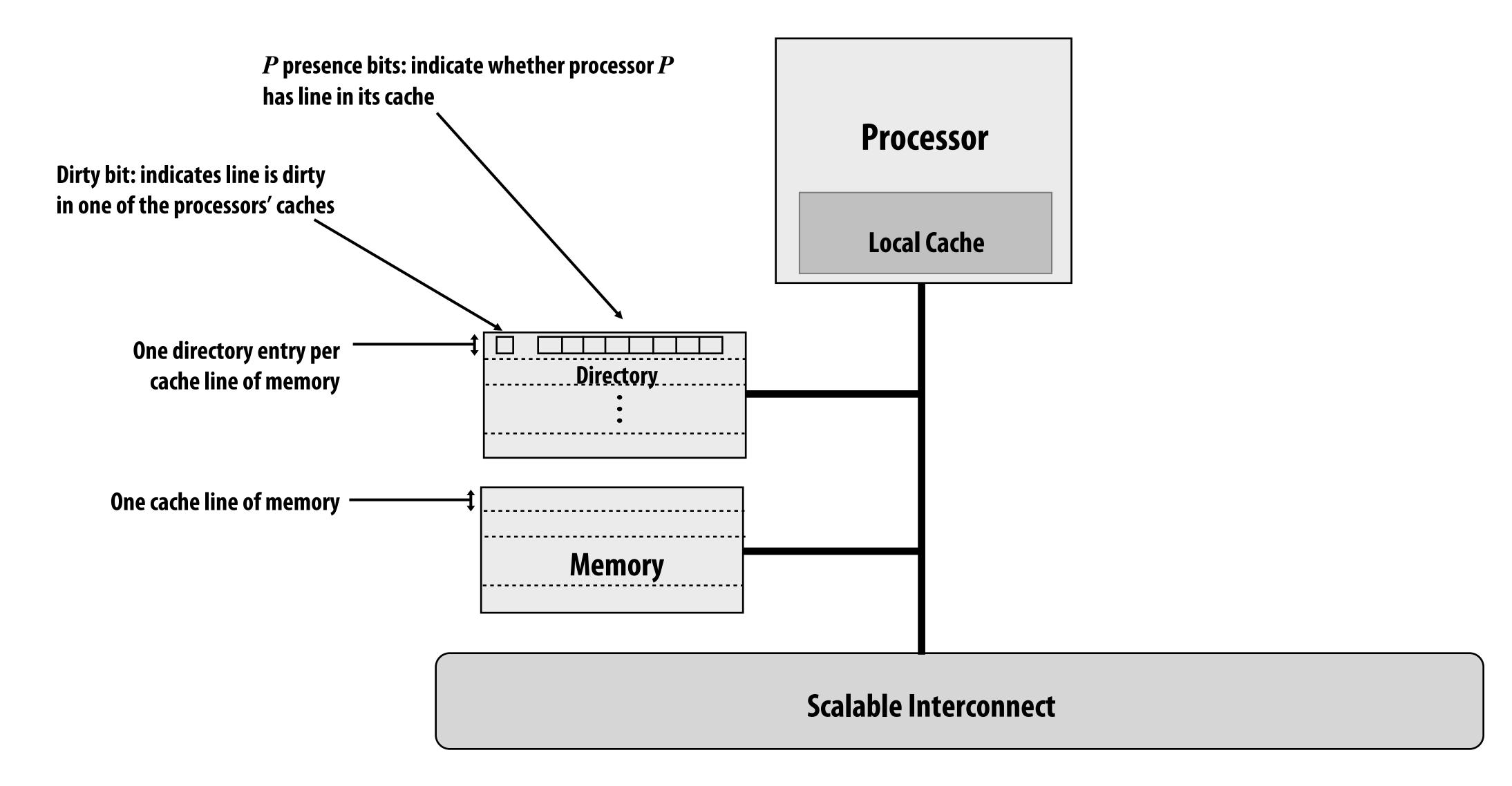


- Multiple rings
 - request
 - snoop
 - ack
 - data (32 bytes)
- Six interconnect nodes: four "slices" of L3 cache + system agent + graphics
- Each bank of L3 connected to ring bus twice

Scalable cache coherence using <u>directories</u>

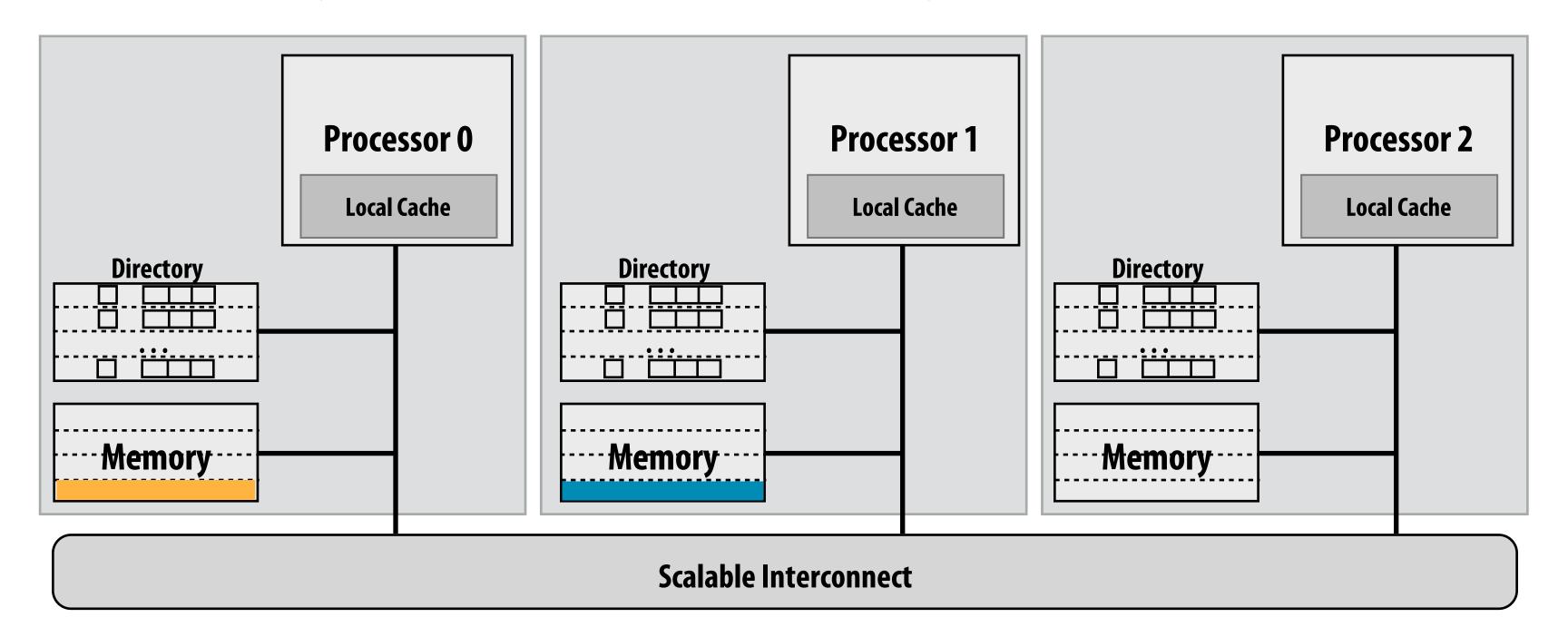
- Snooping schemes <u>broadcast</u> coherence messages to determine the state of a line in the other caches
- Alternative idea: avoid broadcast by storing information about the status of the line in one place: a "directory"
 - The directory entry for a cache line contains information about the state of the cache line in all caches.
 - Caches look up information from the directory as necessary
 - Cache coherence is maintained by point-to-point messages between the caches on a "need to know" basis (not by broadcast mechanisms)
- Still need to maintain invariants
 - SWMR
 - Write serialization

A very simple directory



A distributed directory

Example: directory partition is co-located with memory it describes



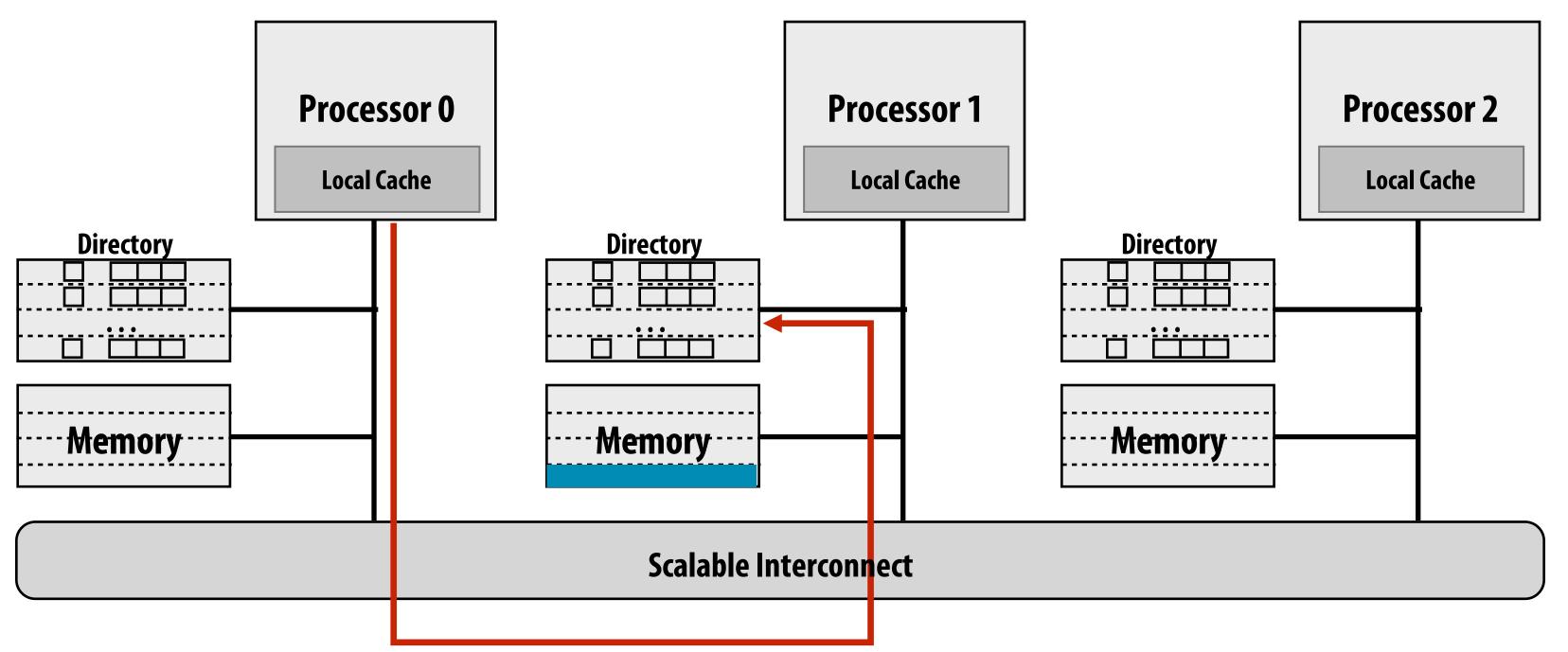
"Home node" of a line: node with memory holding the corresponding data for the line

Example: node 0 is the home node of the yellow line, node 1 is the home node of the blue line

"Requesting node": node containing processor requesting line

Example 1: read miss to clean line

Read from main memory by processor 0 of the blue line: line is not dirty

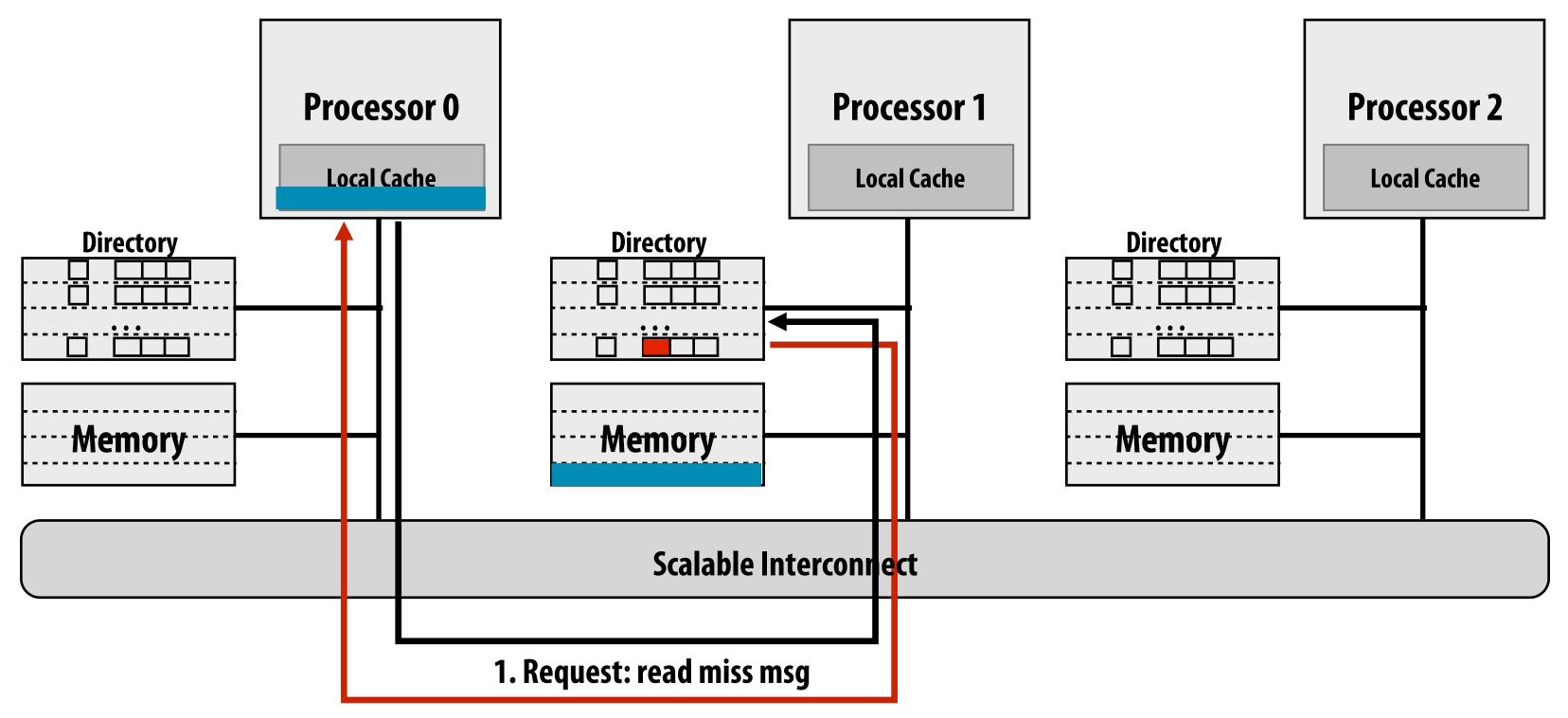


1. Request: read miss msg

- Read miss message sent to home node of the requested line
- Home directory checks entry for line

Example 1: read miss to clean line

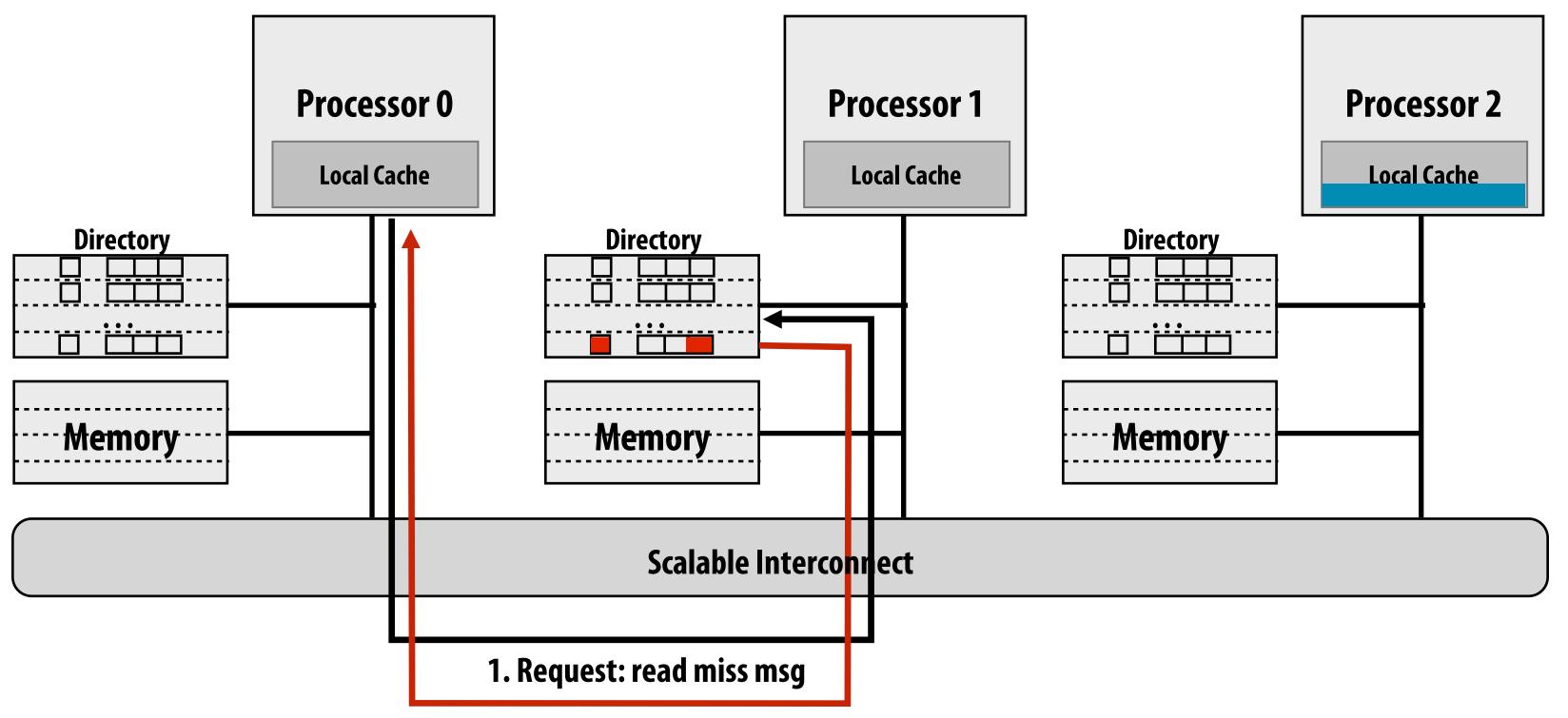
Read from main memory by processor 0 of the blue line: line is not dirty



- 2. Response (line of data from memory)
- Read miss message sent to home node of the requested line
- Home directory checks entry for line
- If dirty bit for cache line is OFF, respond with contents from memory, set presence[0] to true
 (to indicate line is cached by processor 0)

Example 2: read miss to dirty line

Read from main memory by processor 0 of the blue line: line is dirty (contents in P2's cache)

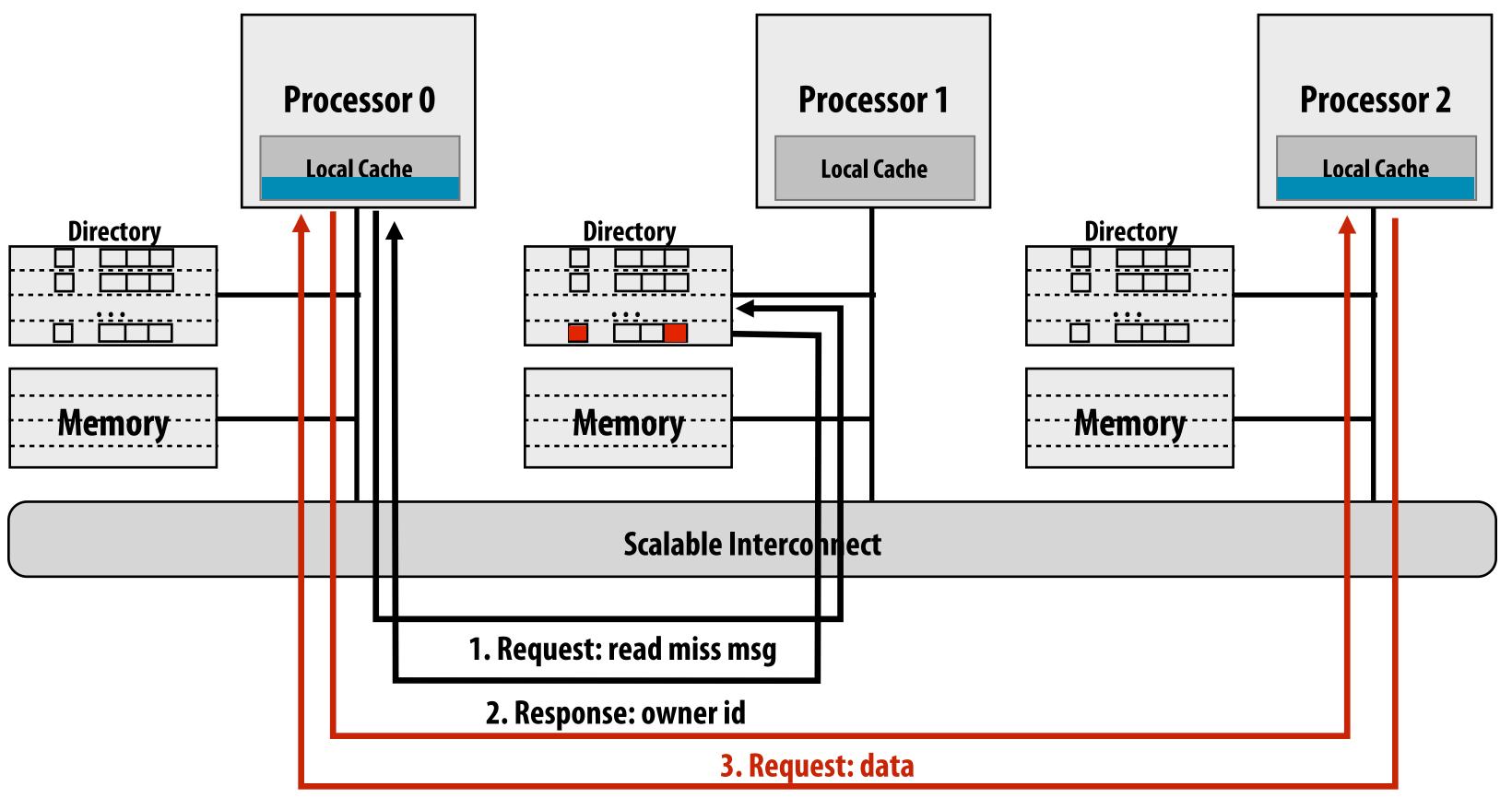


2. Response: owner id

- If dirty bit is ON, then data must be sourced by another processor (with the most up-to-date copy of the line)
- Home node must tell requesting node where to find data
 - Responds with message providing identity of line owner ("get it from P2")

Example 2: read miss to dirty line

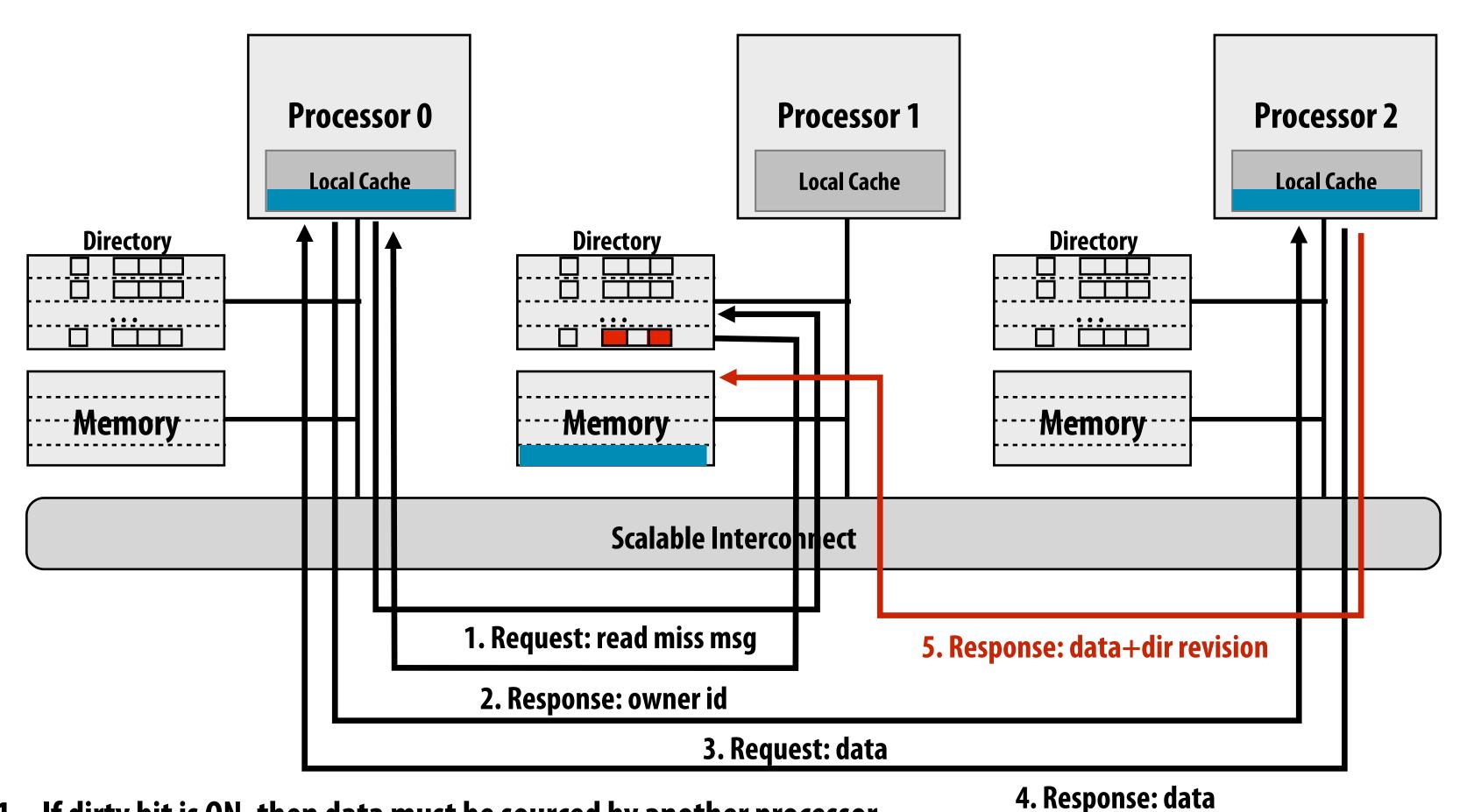
Read from main memory by processor 0 of the blue line: line is dirty (contents in P2's cache)



- 4. Response: data
- 1. If dirty bit is ON, then data must be sourced by another processor
- 2. Home node responds with message providing identity of line owner
- 3. Requesting node requests data from owner
- 4. Owner changes state in cache to SHARED (read only), responds to requesting node

Example 2: read miss to dirty line

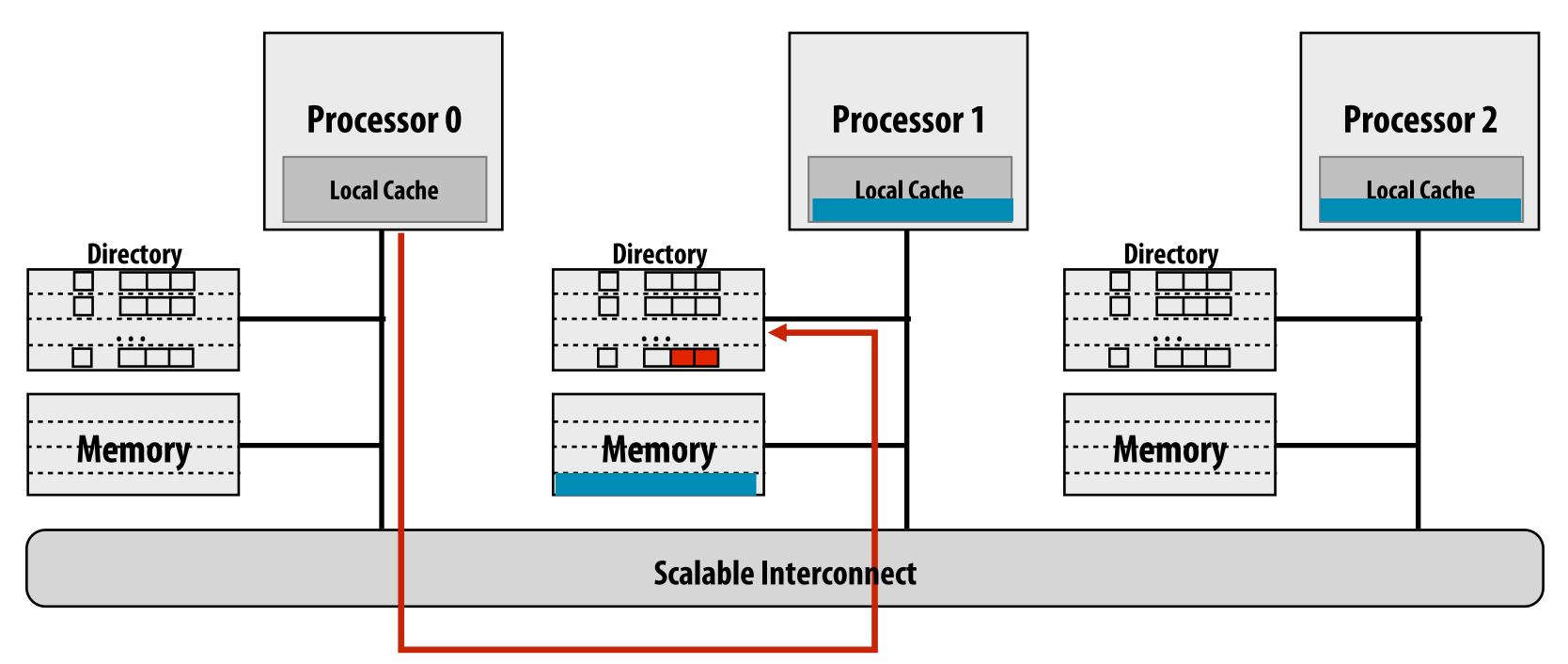
Read from main memory by processor 0 of the blue line: line is dirty (contents in P2's cache)



- 1. If dirty bit is ON, then data must be sourced by another processor
- 2. Home node responds with message providing identity of line owner
- 3. Requesting node requests data from owner
- 4. Owner responds to requesting node, changes state in cache to SHARED (read only)
- 5. Owner also responds to home node, home clears dirty, updates presence bits, updates memory

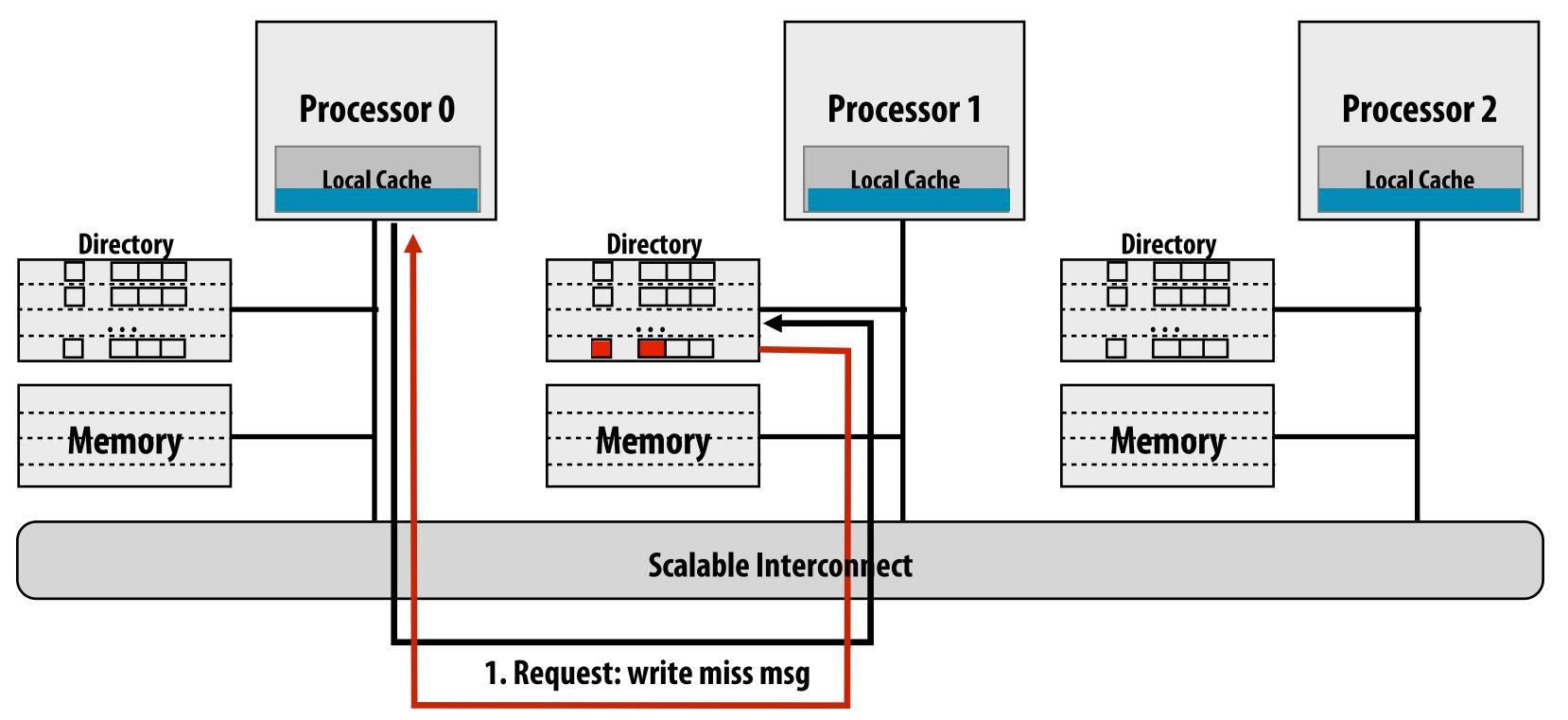
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Write to memory by processor 0: line is clean, but resident in P1's and P2's caches



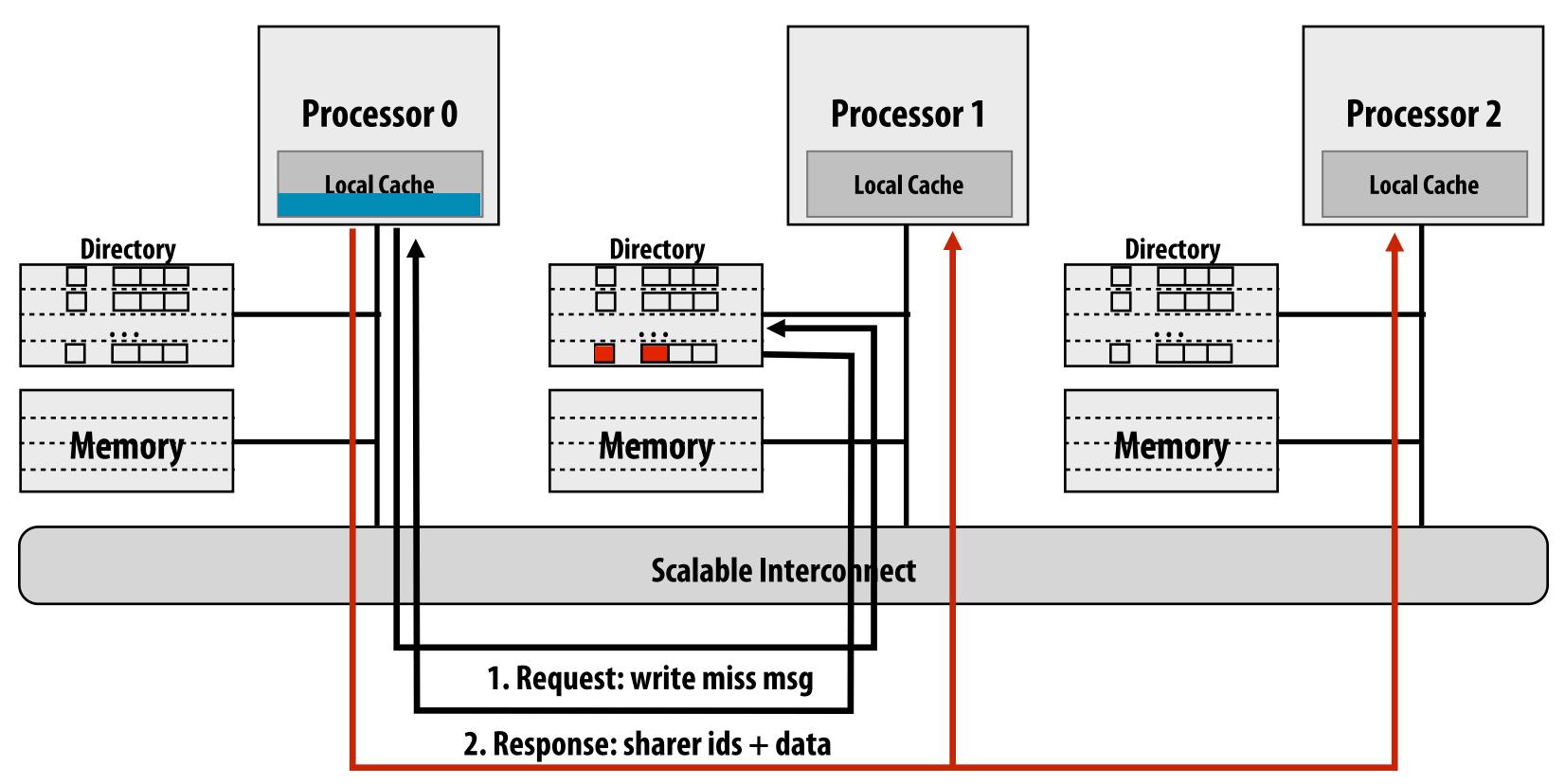
1. Request: write miss msg

Write to memory by processor 0: line is clean, but resident in P1's and P2's caches



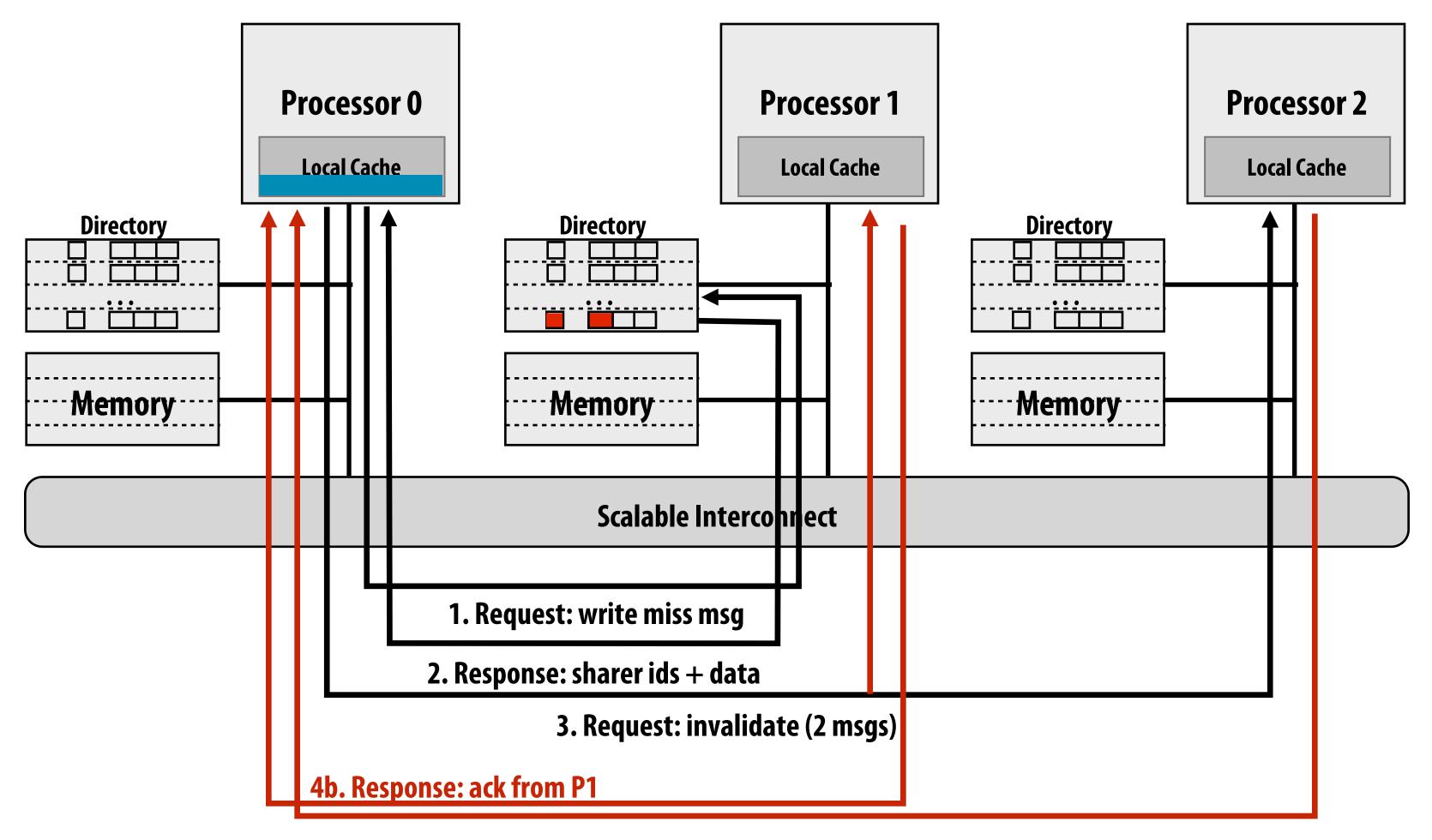
2. Response: sharer ids + data

Write to memory by processor 0: line is clean, but resident in P1's and P2's caches



3. Request: invalidate (2 msgs)

Write to memory by processor 0: line is clean, but resident in P1's and P2's caches



4a. Response: ack from P2

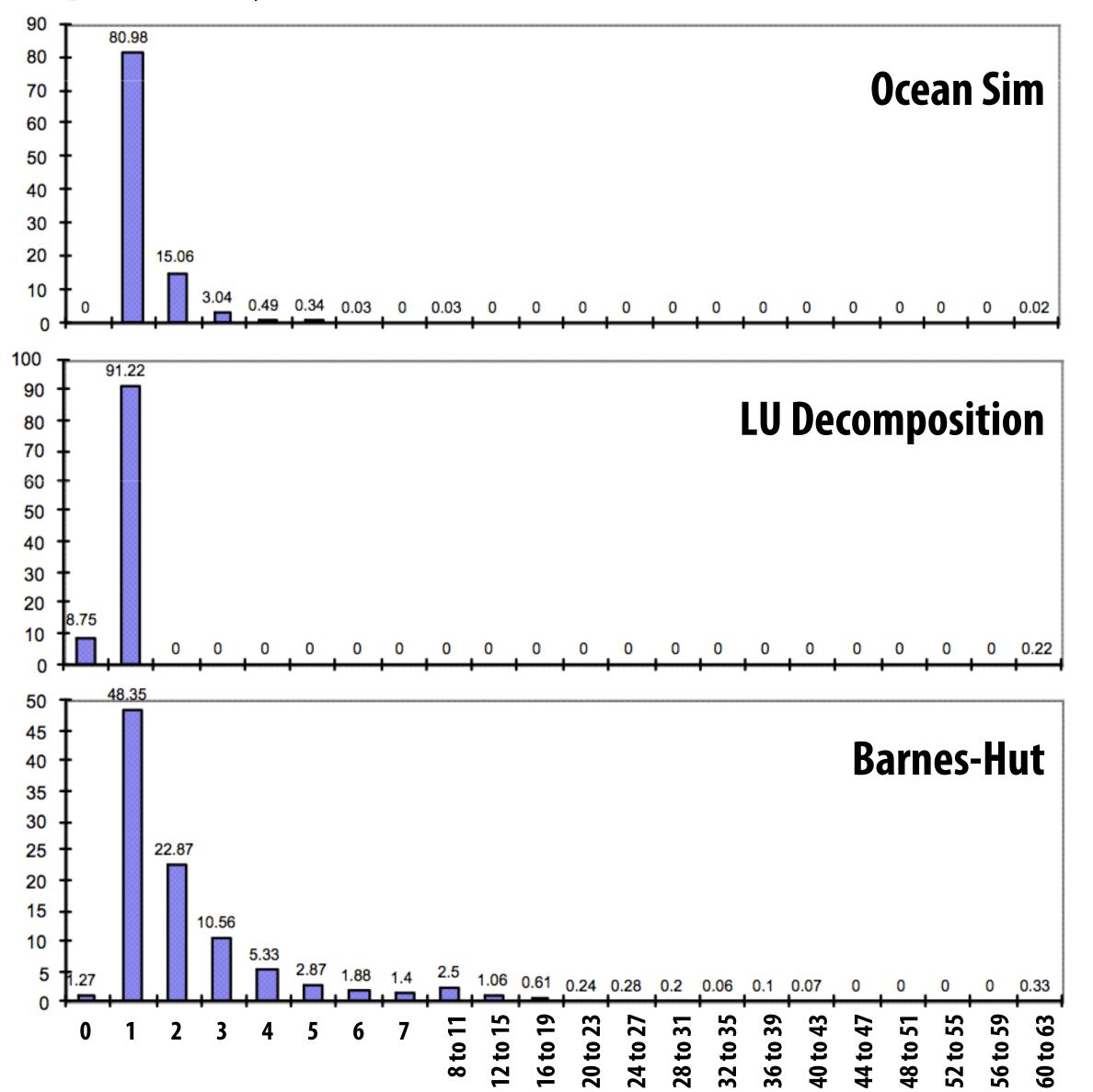
Advantage of directories

- On reads, directory tells requesting node exactly where to get the line from
 - Either from home node (if the line is clean)
 - Or from the owning node (if the line is dirty)
 - Either way, retrieving data involves only point-to-point communication

- On writes, the advantage of directories depends on the number of sharers
 - In the limit, if all caches are sharing data, all caches must be communicated with (just like broadcast in a snooping protocol)
- How are invariants maintained?

Cache invalidation patterns

64 processor system



Graphs plot histogram of number of sharers of a line at the time of a write

In general only a few processors share the line (only a few processors must be told of writes)

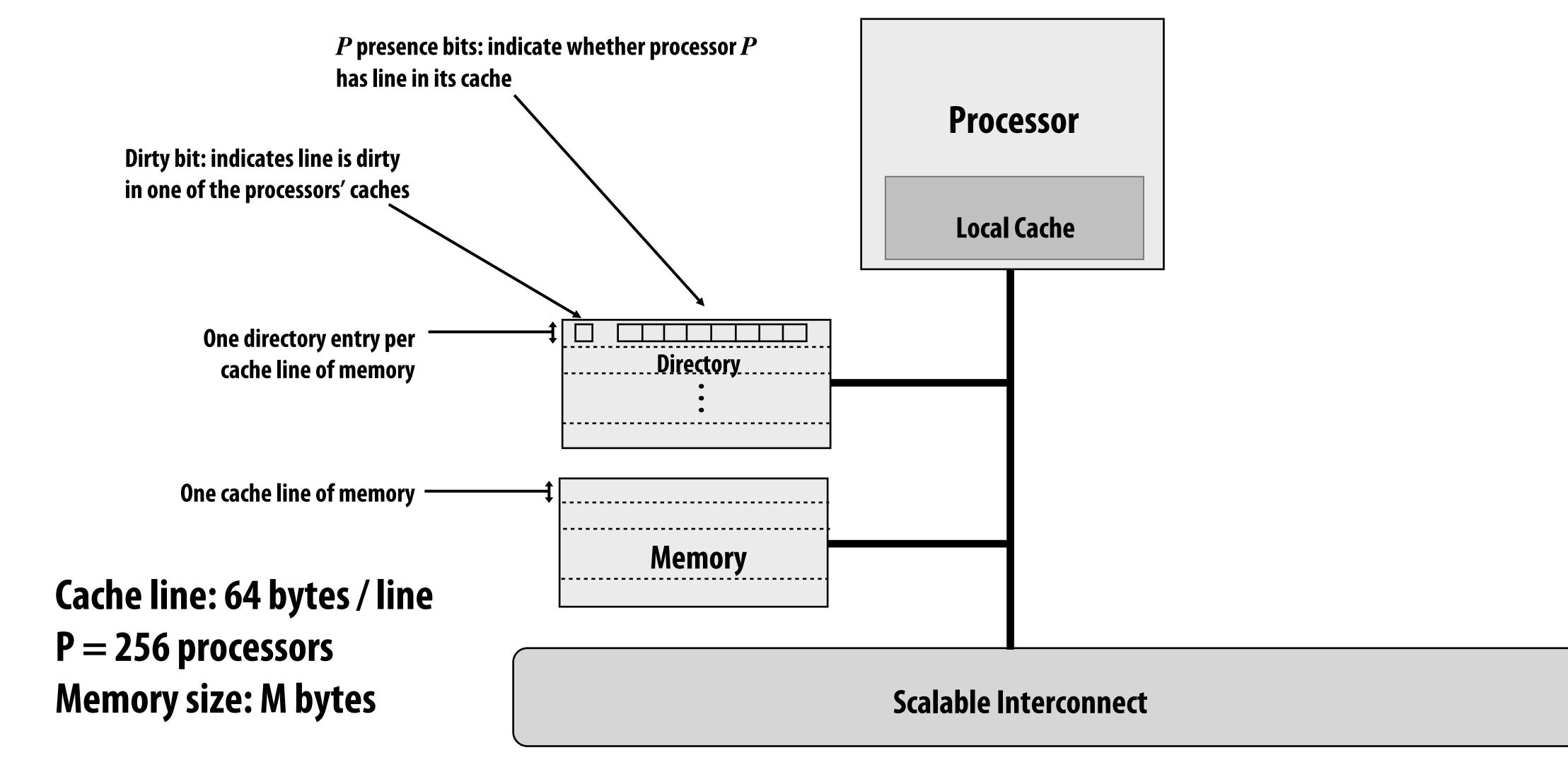
Not shown here, but the expected number of sharers typically increases slowly with P (good!)

In general, only a few sharers during a write

Access patterns

- "Mostly-read" objects: lots of sharers, but writes are infrequent, so communicating with all sharers on a write has minimal impact on performance
- Migratory objects (one processor reads/writes for while, then another, etc.): very few sharers, so count does not scale with number of processors
- Frequently read/written objects: frequent invalidations, but sharer count is low because count cannot build up in short time between invalidations (e.g, shared task queue)
- Low-contention locks: infrequent invalidations, so no performance problem
- High-contention locks: tricky because many readers present when lock released
- Implication 1: directories are useful for limiting coherence traffic
 - Don't need a broadcast mechanism to "tell everyone"
- Implication 2: suggests ways to optimize directory implementations (reduce storage overhead)

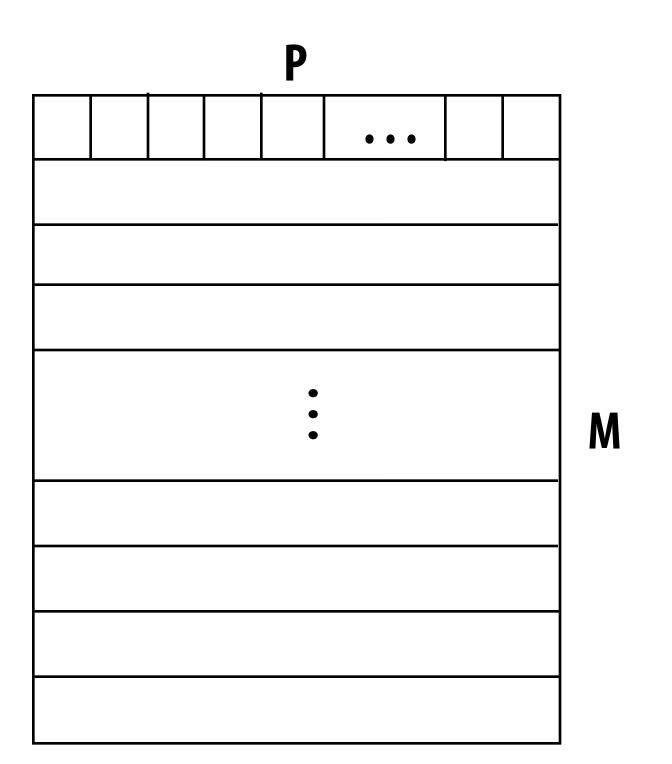
How big is the directory?



How big is the directory?

Full bit vector directory representation

- Recall: one presence bit per node
- Storage proportional to P x M
 - P = number of nodes (e.g., processors)
 - M = number of lines in memory
- Storage overhead rises with P
 - Assume 64 byte cache line size (512 bits)
 - 64 nodes (P=64) \rightarrow 12% overhead
 - 256 nodes (P=256) → 50% overhead
 - 1024 nodes (P=1024) → 200% overhead



Reducing storage overhead of directory

- Optimizations on full-bit vector scheme
 - Increase cache line size (reduce M term)
 - What are possible problems with this approach?

(consider graphs from last lecture)

- Group multiple processors into a single directory "node" (reduce P term)
 - Need only one directory bit per node, not one bit per processor
 - Hierarchical: could use snooping protocol to maintain coherence among processors in a node, directory across nodes
- We will now discuss one alternative scheme
 - Limited pointer schemes (reduce P)

Limited pointer schemes

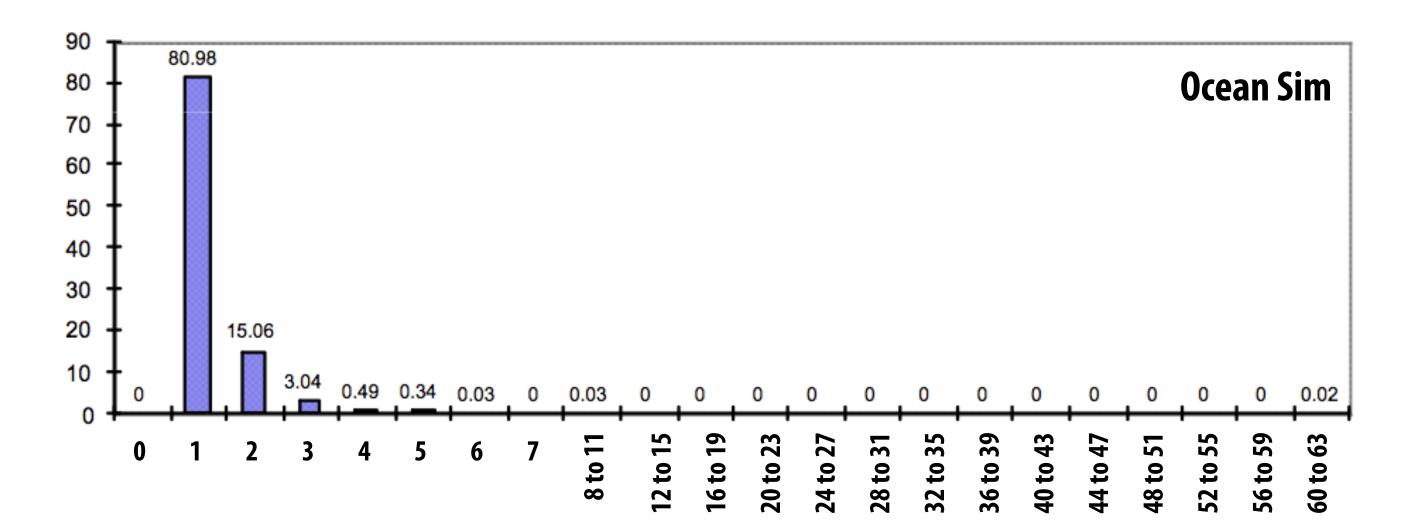
Since data is expected to only be in a few caches at once, storage for a limited number of pointers per directory entry should be sufficient (only need a list of the nodes holding a valid copy of the line!)

Example: 1024 processor system

Full bit vector scheme needs 1024 bits per line

Instead, can store ~100 pointers to nodes holding the line (log₂(1024)=10 bits per pointer)

In practice, our workload evaluation says we can get by with far less than this



Managing overflow in limited pointer schemes

- Fallback to broadcast (if broadcast mechanism exists)
 - When more than max number of sharers, revert to broadcast
- If no broadcast mechanism is present on machine
 - Do not allow more than a max number of sharers
 - On overflow, newest sharer replaces an existing one (must invalidate line in the old sharer's cache)
- Coarse vector fallback
 - Revert to bit vector representation representation
 - Each bit corresponds to K nodes
 - On write, invalidate all nodes a bit corresponds to

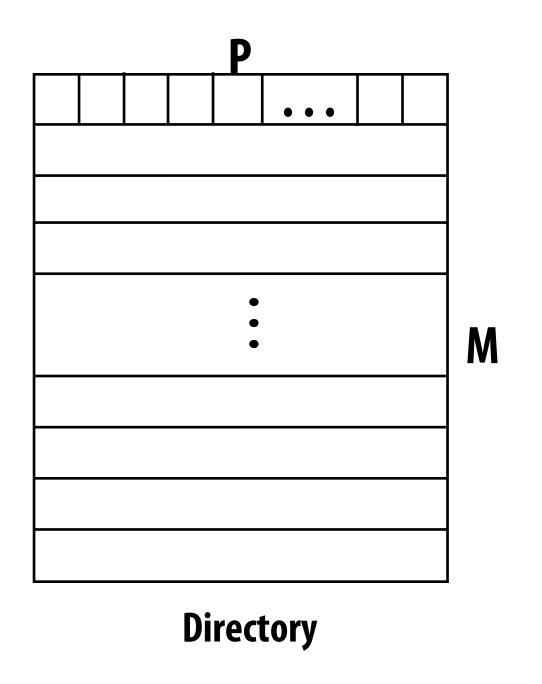
Optimizing for the common case

Limited pointer schemes are a great example of smartly understanding and optimizing for the common case:

- 1. Workload-driven observation: in general the number of cache line sharers is low
- 2. Make the common case simple and fast: array of pointers for first N sharers
- 3. Uncommon case is still handled correctly, just with a slower, more complicated mechanism (the program still works!)
- 4. Extra expense of the complicated solution is tolerable, since it happens infrequently

Limited pointer schemes: summary

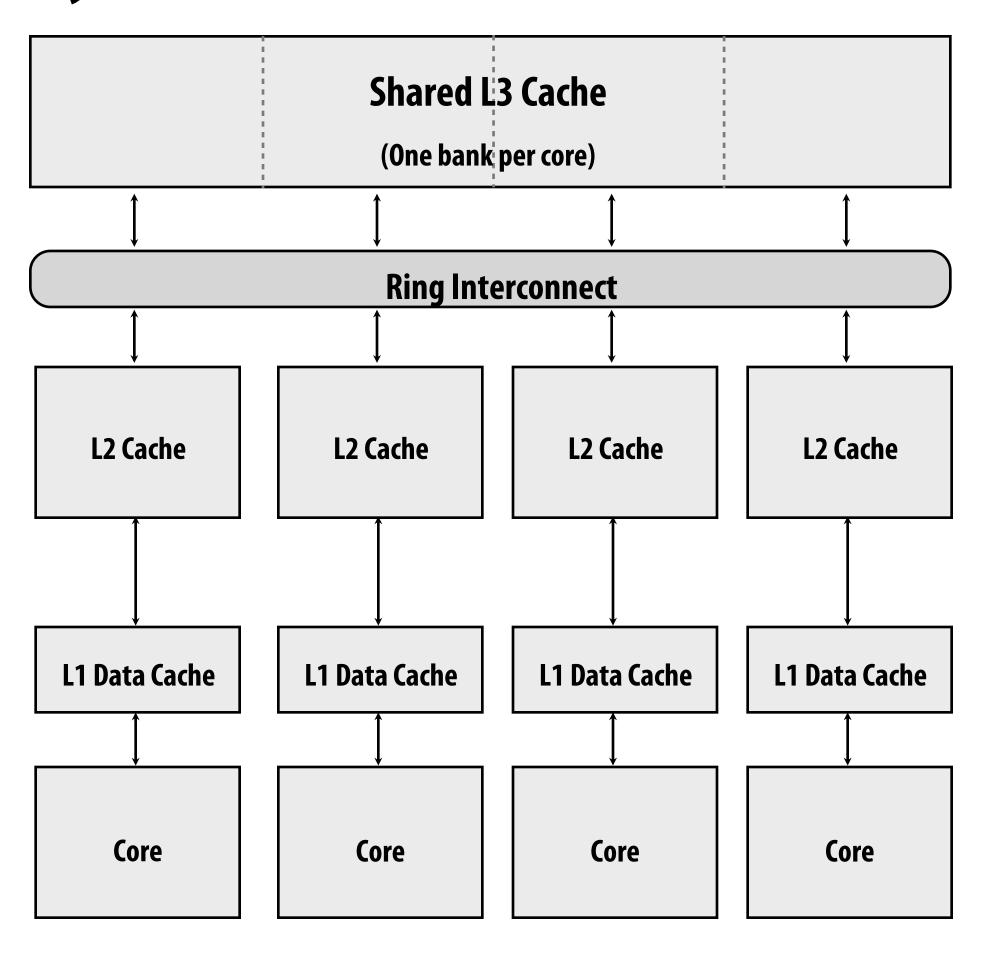
- Limited pointer schemes reduce directory storage overhead caused by large P
 - By adopting a compact representation of a list of sharers
- But do we really even need to maintain a list of sharers for each cache-line-sized chunk of data in memory?



Limiting size of directory

- Key observation: the majority of memory is NOT resident in cache. And to carry out coherence protocol the system only needs sharing information for lines that are currently in cache
 - Most directory entries are empty most of the time
 - 1 MB cache, 1 GB memory per node \rightarrow 99.9% of directory entries are empty

Directory coherence in Intel Core i7 CPU



 L3 serves as centralized directory for all lines in the L3 cache

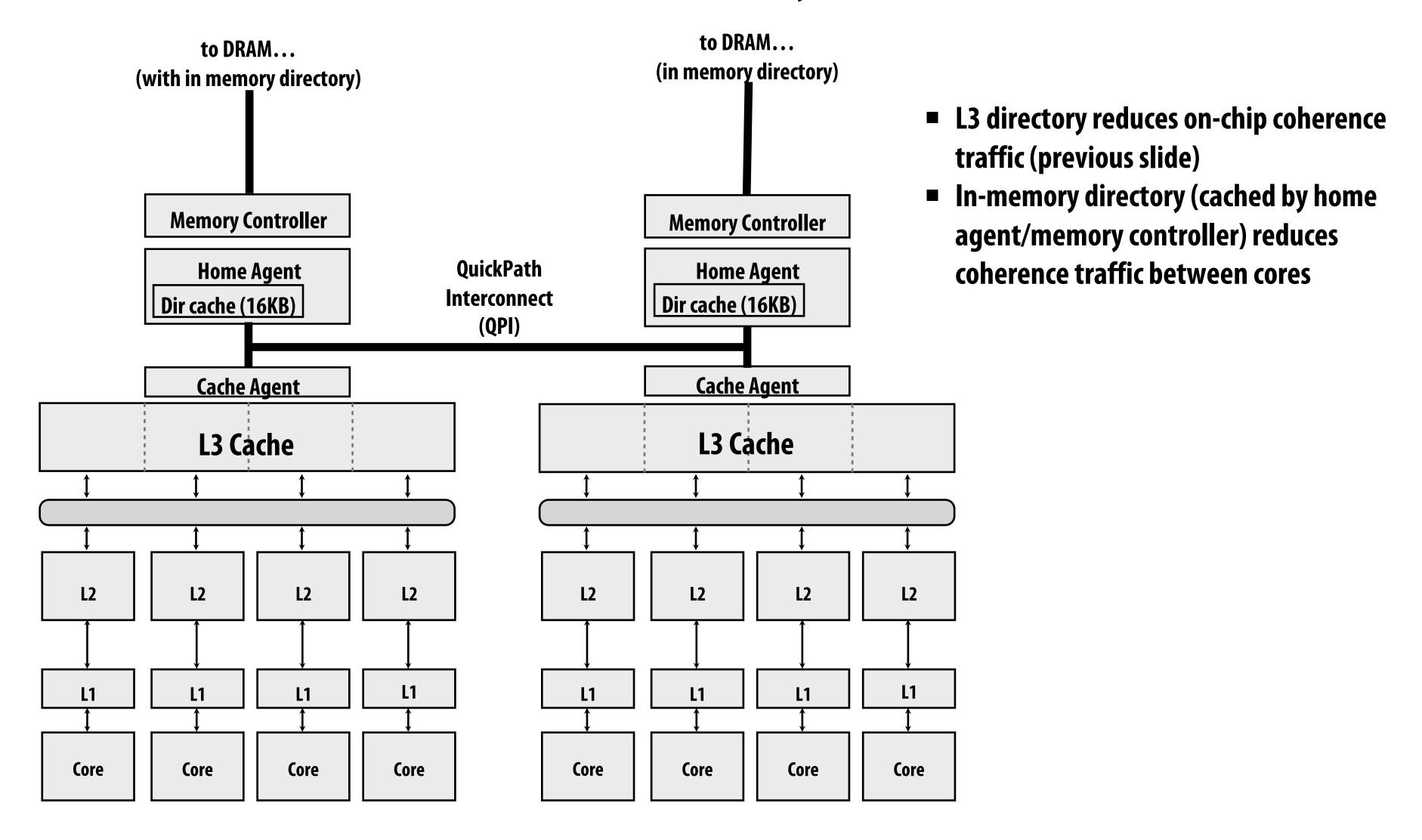
(Since L3 is an inclusive cache, any line in L2 is guaranteed to also be resident in L3)

- Directory maintains list of L2 caches containing line
- Instead of broadcasting coherence traffic to all L2's, only send coherence messages to L2's that contain the line

(Core i7 interconnect is a ring, it is not a bus)

- Directory dimensions:
 - P=4
 - M = number of L3 cache lines

Coherence in multi-socket Intel systems



Summary: directory-based coherence

- Primary observation: broadcast doesn't scale, but luckily we don't need broadcast to ensure coherence because often the number of caches containing a copy of a line is small
- Instead of snooping, just store the list of sharers in a "directory" and check the list as necessary

- One challenge: reducing overhead of directory storage
 - Use hierarchies of processors or larger line sizes
 - Limited pointer schemes: exploit fact the most processors not sharing line
 - Exploit fact that most lines are not in cache
 - Implementations can get very complex

Implementing Synchronization

Now that you understand implementations of cache coherence, the cost of implementing synchronization primitives on a modern machine will become very apparent.

Three phases of a synchronization event

- 1. Acquire method
 - How a thread attempts to gain access to lock
- 2. Waiting algorithm
 - How a thread waits for access to be granted to lock
- 3. Release method
 - How thread enables other threads to gain lock when its work in the synchronized region is complete

Busy waiting

Busy waiting (a.k.a. "spinning")

```
while (condition X not true) {}
logic that assumes X is true
```

- In classes like CS107/CS110 or in operating systems, you have certainly also talked about synchronization
 - You might have been taught busy-waiting is bad: why?

"Blocking" synchronization

Idea: if progress cannot be made because a lock cannot be acquired, it is desirable to free up execution resources for another thread (preempt the running thread)

pthreads mutex example

```
pthread_mutex_t mutex;
pthread_mutex_lock(&mutex);
```

Busy waiting vs. blocking

Busy-waiting can be preferable to blocking if:

- Scheduling overhead is larger than expected wait time
- A processor's resources not needed for other tasks
 - This is often the case in a parallel program since we usually don't oversubscribe
 a system when running a performance-critical parallel app (e.g., there aren't
 multiple CPU-intensive programs running at the same time)
 - Clarification: be careful to not confuse the above statement with the value of multi-threading (interleaving execution of multiple threads/tasks to hiding long latency of memory operations) with other work within the same app.

Examples:

Implementing Locks

Warm up: a simple, but incorrect, lock

Problem: data race because LOAD-TEST-STORE is not atomic!

Processor 0 loads address X, observes 0
Processor 1 loads address X, observes 0
Processor 0 writes 1 to address X
Processor 1 writes 1 to address X

Test-and-set based lock

Atomic test-and-set instruction:

Test-and-set lock: consider coherence traffic

Processor 1 Processor 2 Processor 3 Invalidate line Invalidate line BusRdX Update line in cache (set to 1) Invalidate line BusRdX Attempt to update (t&s fails) **Invalidate line BusRdX** Attempt to update (t&s fails) [P1 is holding lock...] T&S **Invalidate line** BusRdX Attempt to update (t&s fails) **Invalidate line** T&S BusRdX Attempt to update (t&s fails) BusRdX **Invalidate line Update line in cache (set to 0)** T&S **Invalidate line BusRdX Update line in cache (set to 1)** = thread has lock

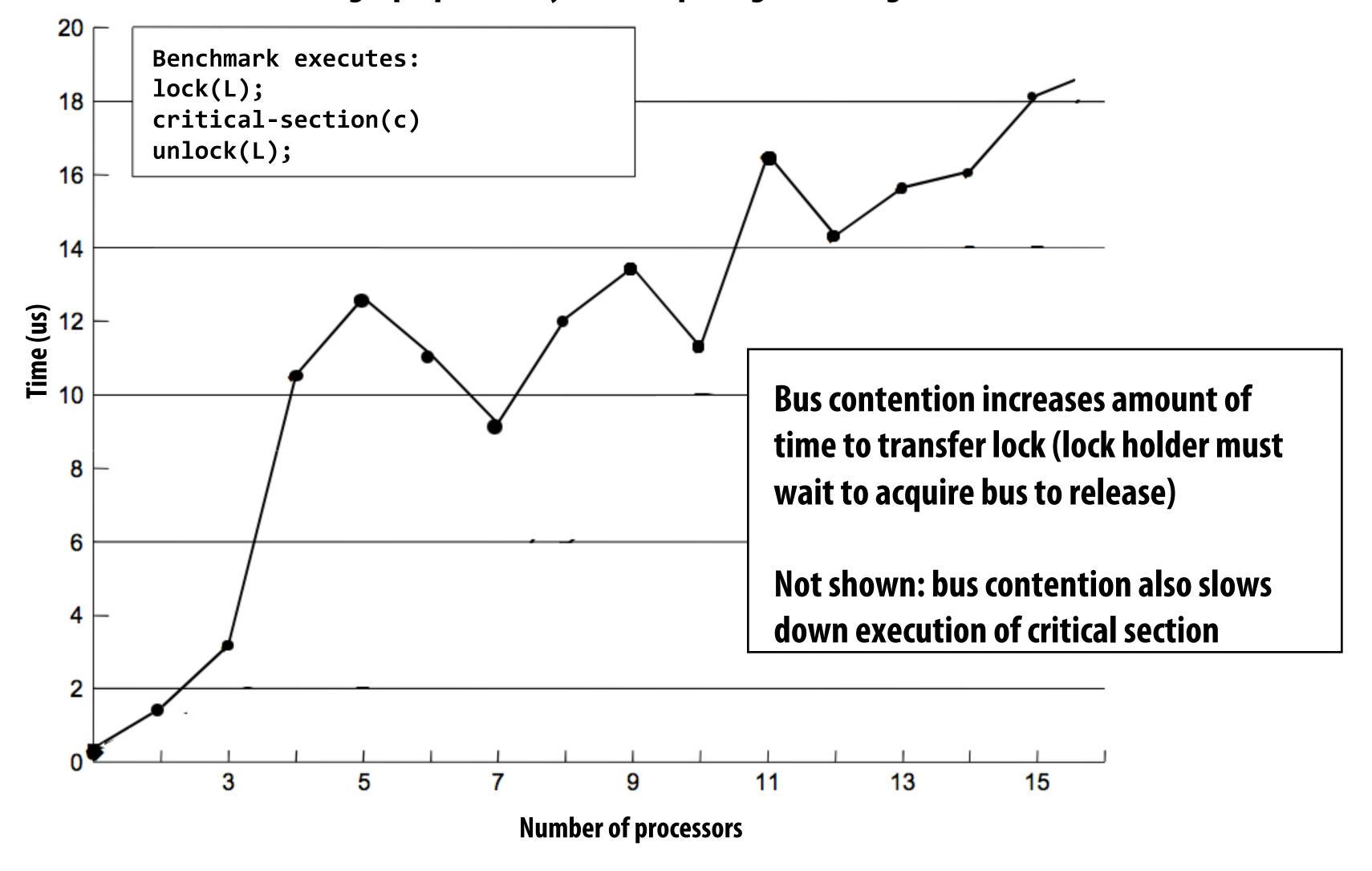
Check your understanding

On the previous slide, what is the duration of time the thread running on P1 holds the lock?

At what points in time does P1's cache contain a valid copy of the cache line containing the lock variable?

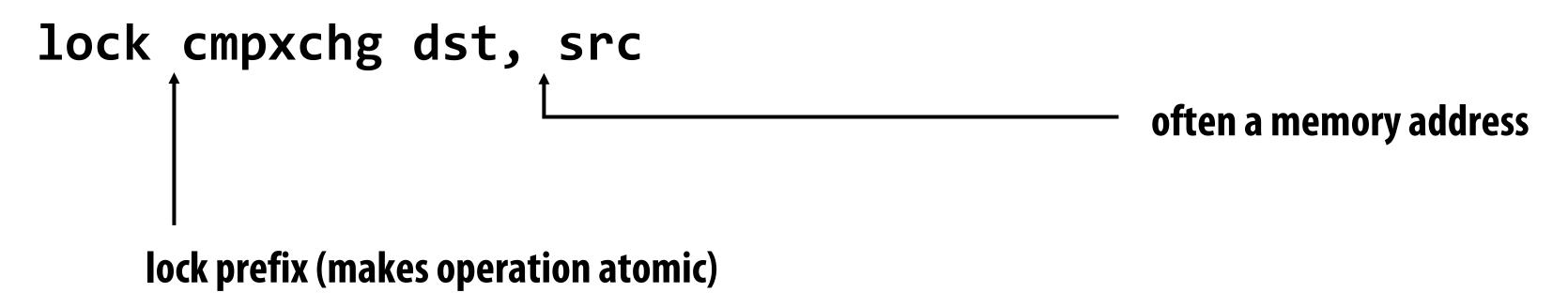
Test-and-set lock performance

Benchmark: execute a total of N lock/unlock sequences (in aggregate) by P processors Critical section time removed so graph plots only time acquiring/releasing the lock



x86 cmpxchg

Compare and exchange (atomic when used with lock prefix)



```
x86 accumulator register

if (dst == EAX)

ZF = 1 ← flag register

dst = src

else

ZF = 0

EAX = dst

x86 accumulator register

flag register

self-ch

atomic

bool of

if
```

Self-check: Can you implement assembly for atomic compare-and-swap using cmpxchg?

```
bool compare_and_swap(int* x, int a, int b) {
   if (*x == a) {
      *x = b;
      return true;
   }
   return false;
}
```

Desirable lock performance characteristics

Low latency

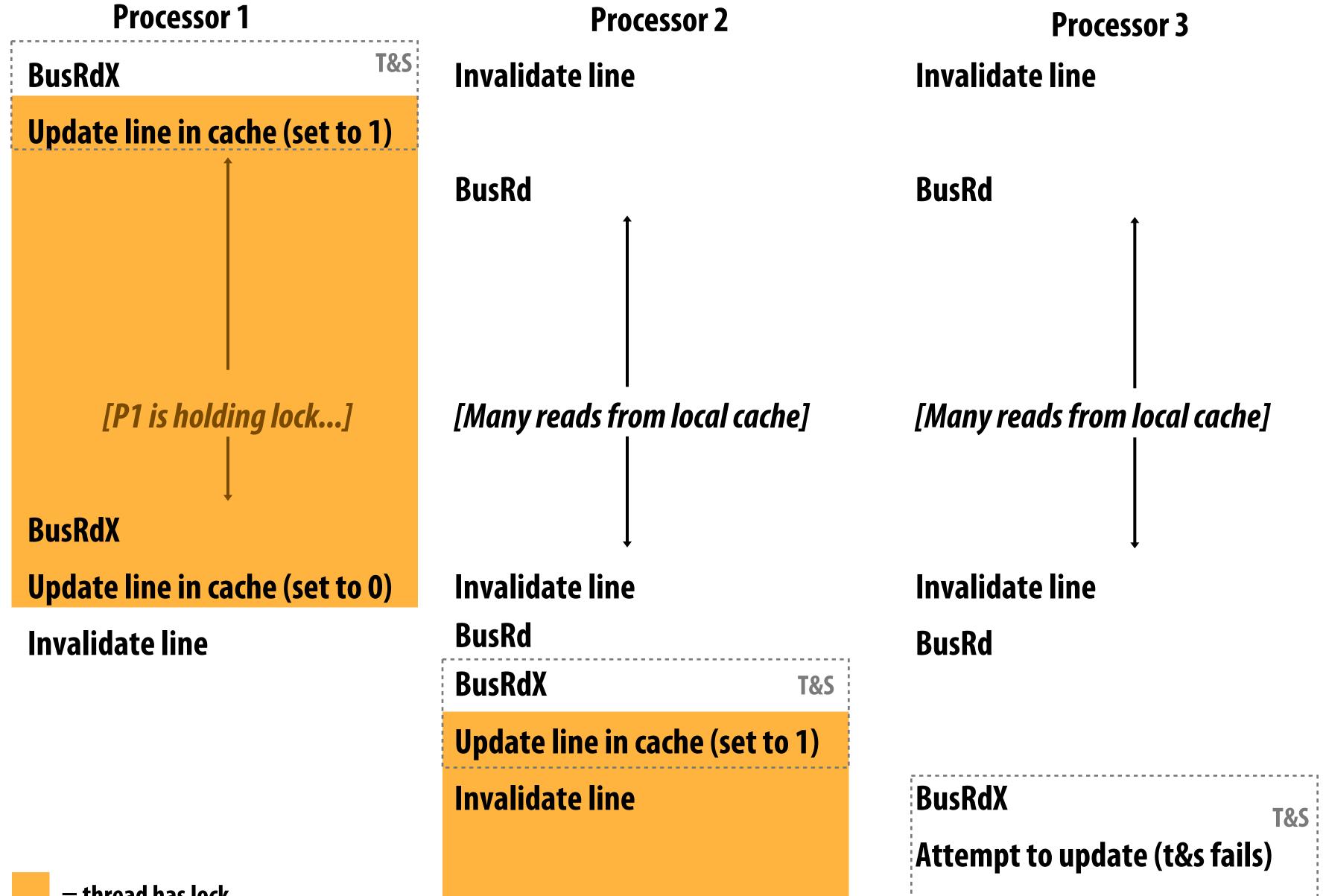
- If lock is free and no other processors are trying to acquire it, a processor should be able to acquire the lock quickly
- Low interconnect traffic
 - If all processors are trying to acquire lock at once, they should acquire the lock in succession with as little traffic as possible
- Scalability
 - Latency / traffic should scale reasonably with number of processors
- Low storage cost
- Fairness
 - Avoid starvation or substantial unfairness
 - One ideal: processors should acquire lock in the order they request access to it

Simple test-and-set lock: low latency (under low contention), high traffic, poor scaling, low storage cost (one int), no provisions for fairness

Test-and-test-and-set lock

```
void Lock(int* lock) {
  while (1) {
                                    // while another processor has the lock...
    while (*lock != 0);
                                    // (assume *lock is NOT register allocated)
    if (test_and_set(*lock) == 0) // when lock is released, try to acquire it
      return;
void Unlock(int* lock) {
   *lock = 0;
```

Test-and-test-and-set lock: coherence traffic



Test-and-test-and-set characteristics

- Slightly higher latency than test-and-set in <u>uncontended</u> case
 - Must test... then test-and-set
- Generates much less interconnect traffic
 - One invalidation, per waiting processor, per lock release (O(P) invalidations)
 - This is O(P²) interconnect traffic if all processors have the lock cached
 - Recall: test-and-set lock generated one invalidation per waiting processor <u>per test</u>
- More scalable (due to less traffic)
- Storage cost unchanged (one int)
- Still no provisions for fairness

Test-and-set lock with back off

Upon failure to acquire lock, delay for awhile before retrying

```
void Lock(volatile int* 1) {
  int amount = 1;
  while (1) {
   if (test_and_set(*1) == 0)
      return;
   delay(amount);
   amount *= 2;
  }
}
```

- Same <u>uncontended</u> latency as test-and-set, but potentially higher latency under contention. Why?
- Generates less traffic than test-and-set (not continually attempting to acquire lock)
- Improves scalability (due to less traffic)
- Storage cost unchanged (still one int for lock)
- Exponential back-off can cause severe unfairness
 - Newer requesters back off for shorter intervals

Ticket lock

Main problem with test-and-set style locks: upon release, all waiting processors attempt to acquire lock using test-and-set



```
struct lock {
   int next_ticket;
   int now_serving;
};

void Lock(lock* 1) {
   int my_ticket = atomic_increment(&1->next_ticket);  // take a "ticket"
   while (my_ticket != 1->now_serving);  // wait for number
}

void unlock(lock* 1) {
   l->now_serving++;
}
```

No atomic operation needed to acquire the lock (only a read)
Result: only one invalidation per lock release (O(P) interconnect traffic)

Array-based lock

Each processor spins on a different memory address
Utilizes atomic operation to assign address on attempt to acquire

```
struct lock {
   padded_int status[P];  // padded to keep off same cache line
   int head;
};
int my_element;
void Lock(lock* 1) {
 my_element = atomic_circ_increment(&l->head);  // assume circular increment
 while (l->status[my_element] == 1);
void unlock(lock* 1) {
 1->status[my_element] = 1;
 1->status[circ_next(my_element)] = 0;
                                                  // next() gives next index
```

O(1) interconnect traffic per release, but lock requires space linear in P Also, the atomic circular increment is a more complex operation (higher overhead)

Additional atomic operations

Atomic operations provided by CUDA

```
atomicAdd(int* address, int val);
int
float atomicAdd(float* address, float val);
     atomicSub(int* address, int val);
int
     atomicExch(int* address, int val);
int
float atomicExch(float* address, float val);
int
     atomicMin(int* address, int val);
int
     atomicMax(int* address, int val);
unsigned int atomicInc(unsigned int* address, unsigned int val);
unsigned int atomicDec(unsigned int* address, unsigned int val);
int
     atomicCAS(int* address, int compare, int val);
int
     atomicAnd(int* address, int val); // bitwise
     atomicOr(int* address, int val); // bitwise
int
int
     atomicXor(int* address, int val); // bitwise
```

(omitting additional 64 bit and unsigned int versions)

Implementing atomic fetch-and-op

```
// atomicCAS:
// atomic compare and swap performs the following logic atomically
int atomicCAS(int* addr, int compare, int val) {
   int old = *addr;
   *addr = (old == compare) ? val : old;
   return old;
}
```

Exercise: how can you build an atomic fetch+op out of atomicCAS()?

Example: atomic_min()

```
int atomic_min(int* addr, int x) {
   int old = *addr;
   int new = min(old, x);
   while (atomicCAS(addr, old, new) != old) {
      old = *addr;
      new = min(old, x);
   }
}
```

What about these operations?

```
int atomic_increment(int* addr, int x);  // for signed values of x
void lock(int* addr);
```

Load-linked, store conditional (LL/SC)

- Pair of corresponding instructions (not a single atomic instruction like compare-and-swap)
 - load_linked(x): load value from address
 - store_conditional(x, value): store value to x, if x hasn't been written to since corresponding LL
- Corresponding ARM instructions: LDREX and STREX
- How might LL/SC be implemented on a cache coherent processor?

Coming up...

- Imagine you have a shared variable for which contention is low.
 So it is <u>unlikely</u> that two processors will enter the critical section at the same time?
- You could hope for the best, and avoid the overhead of taking the lock since it is likely that mechanisms for ensuring mutual exclusion are not needed for correctness
 - Take a "optimize-for-the-common-case" attitude
- What happens if you take this approach and you're wrong: in the middle of the critical region, another process enters the same region?

Preview: transactional memory

```
atomic
{    // begin transaction

perform atomic computation here ...
}    // end transaction
```

Instead of ensuring mutual exclusion via locks, system will proceed as if no synchronization was necessary. (it speculates!)

System provides hardware/software support for "rolling back" all loads and stores in the critical region if it detects (at run-time) that another thread has entered same region at the same time.

Loop Parallelism (LLP)

- Overwhelming majority of scientific/engineering applications are expressed in terms of iterative constructs, that is, loops
 - Focus on parallelizing loops

- Particular useful approach if starting from an existing program
 - Major restructuring is impractical/unnecessary

- Goal of exploiting LLP is to evolve the sequential program into a parallel program
 - Through transformations that leave the program semantics unchanged

LLP works well for shared address space (e.g. Multicore)

General Approach for Loop Parallelism

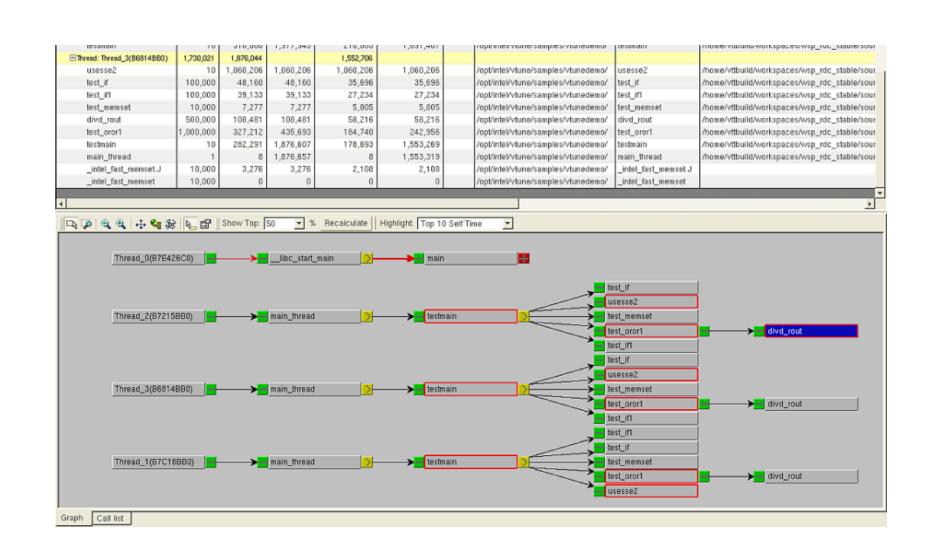
Find the hotspots Eliminate loop-carried dependencies Parallelize the loops Optimize the loop schedule

Find the Hotspots

By code inspection



By using performance analysis tools



Parallel Loops

```
for (i = 0; i < n; i++) {
    A[i] = A[i] + B;
}</pre>
```

```
for (i = 1; i < n; i++) {
    A[i] = A[i-1] + C[i-1];    /* S1 */
    B[i] = B[i-1] + A[i];    /* S2 */
}</pre>
```

Parallel Loops

```
for (i = 0; i < n; i++) {
    A[i] = A[i] + B[i]; /* S1 */
    B[i+1] = C[i] + D[i]; /* S2 */
}</pre>
```

Data Parallelism with OpenMP

For-loop with independent iterations

```
for (i = 0; i < n; i++)
c[i] = a[i] + b[i];
```

For-loop parallelized using an OpenMP pragma

```
% cc -xopenmp source.c
% setenv OMP_NUM_THREADS 4
% a.out
gcc source.c -fopenmp
```

Privatizing Variables

- Critical to performance!
- OpenMP pragmas:
 - Designed to make parallelizing sequential code easier
 - Makes copies of "private" variables automatically
 - And performs some automatic initialization, too
 - Must specify shared/private per-variable in parallel region
 - private: Uninitialized private data
 - Private variables are undefined on entry and exit of the parallel region
 - shared: All-shared data
 - threadprivate: "static" private for use across several parallel regions

Firstprivate/Lastprivate Clauses

- firstprivate (list)
 - All variables in the list are initialized with the value the original object had before entering the parallel region

lastprivate(list)

- The thread that executes the last iteration or section in sequential order updates the value of the objects in the list

Example Private Variables

```
main()
 A = 10;
#pragma omp parallel
 #pragma omp for private(i) firstprivate(A) lastprivate(B)...
  for (i=0; i<n; i++)
      . . . .
                       /*-- A undefined, unless declared
      B = A + i;
                            firstprivate --*/
      . . . .
                       /*-- B undefined, unless declared
                            lastprivate --*/
       End of OpenMP parallel region --*/
```

for directive Example

```
#pragma omp parallel default(none) \
        shared(n,a,b,c,d) private(i)
    #pragma omp for
     for (i=0; i< n-1; i++)
         b[i] = (a[i] + a[i+1])/2;
    #pragma omp for
     for (i=0; i<n; i++)
         d[i] = 1.0/c[i];
    /*-- End of parallel region --*/
                         (implied barrier)
```

Nested Loop Parallelism

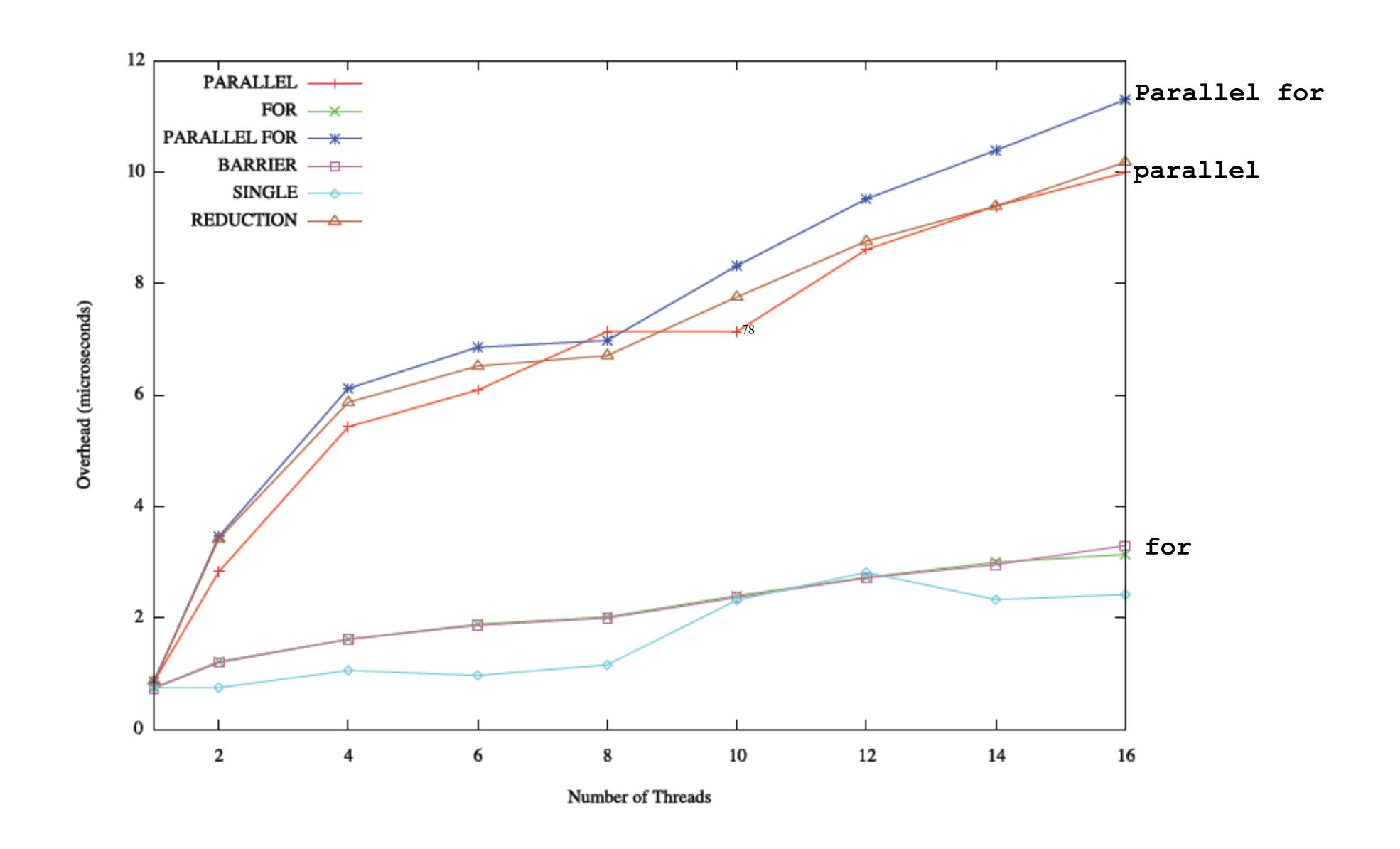
```
#pragma omp parallel for
for(int y=0; y<25; ++y)
{
    #pragma omp parallel for
    for(int x=0; x<80; ++x)
        tick(x,y);
}</pre>
```

Multiple Part Parallel Regions

- You can also have a "multi-part" parallel region
 - Allows easy alternation of serial & parallel parts
 - Doesn't require re-specifying # of threads, etc.

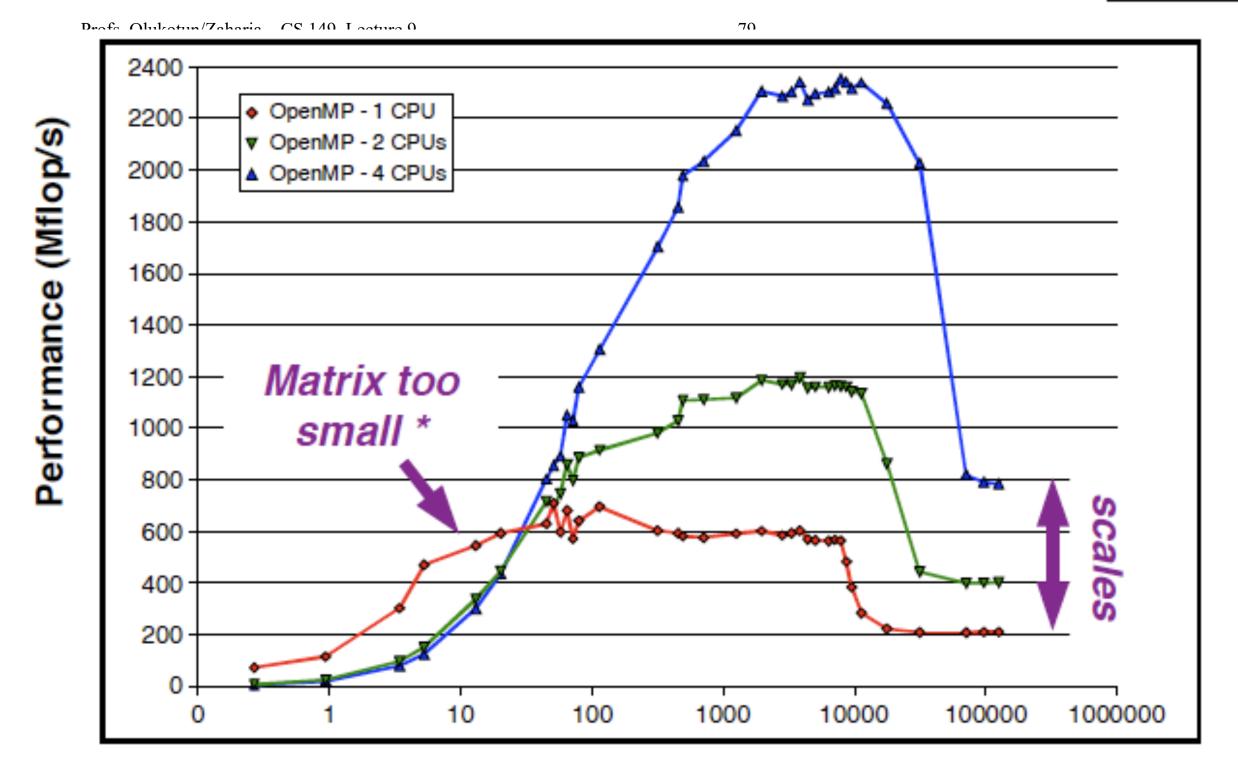
```
#pragma omp parallel . . .
{
    #pragma omp for
    . . . Loop here . . .
    #pragma omp single
    . . . Serial portion here . . .
    #pragma omp sections
    . . . Sections here . . .
}
```

OMP Directives Overheads



"if" Clause

- if (scalar expression)
 - Only execute in parallel if expression evaluates to true
 - Otherwise, execute serially



Performance without if clause

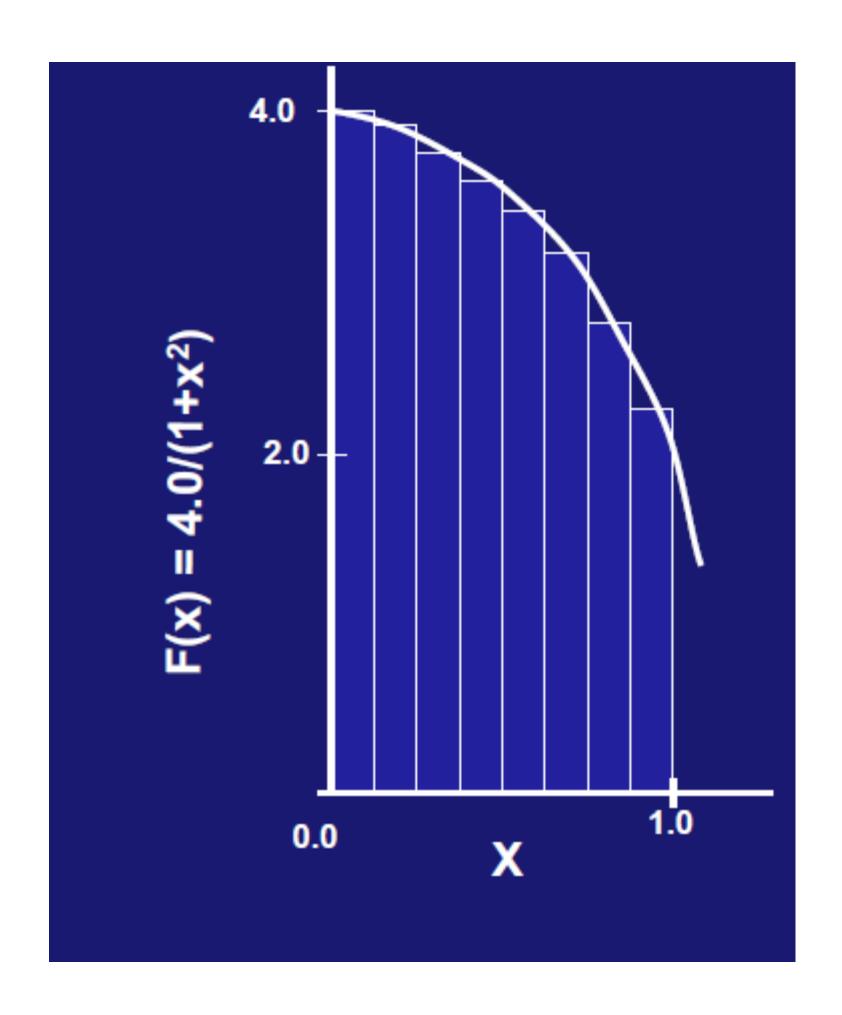
Memory Footprint (KByte)

Reductions in OpenMP

- May add reduction clause to parallel for pragma
- Specify reduction operation and reduction variable
- OpenMP takes care of storing partial results in private variables and combining partial results after the loop
 Profs. Olukotun/Zaharia CS 149 Lecture 9
- The reduction clause has this syntax: reduction (<op> :<variable>)
- Operators

```
- + Sum
- * Product
- & , | , ^ Bitwise and, or , exclusive or
- & & , | | Logical and, or
```

Example: Numerical Integration



We know mathematically that

$$\pi = \int_0^1 \frac{4.0}{(1+x^2)} \, dx$$

$$\sum_{i=0}^{N} F(x_i) \Delta x \approx \pi$$

We can approximate the integral as a sum of rectangles:

Sequential Pi Computation

```
static long num_steps = 100000;
double step;

void main () {
   int i; double x, pi, sum = 0.0;
   step = 1.0/(double) num_steps;
   for (i=0;i< num_steps; i++){
      x = (i+0.5)*step;
      sum = sum + 4.0/(1.0+x*x);
   }
   pi = step * sum;
}</pre>
```

Loop Parallelized Pi Computation

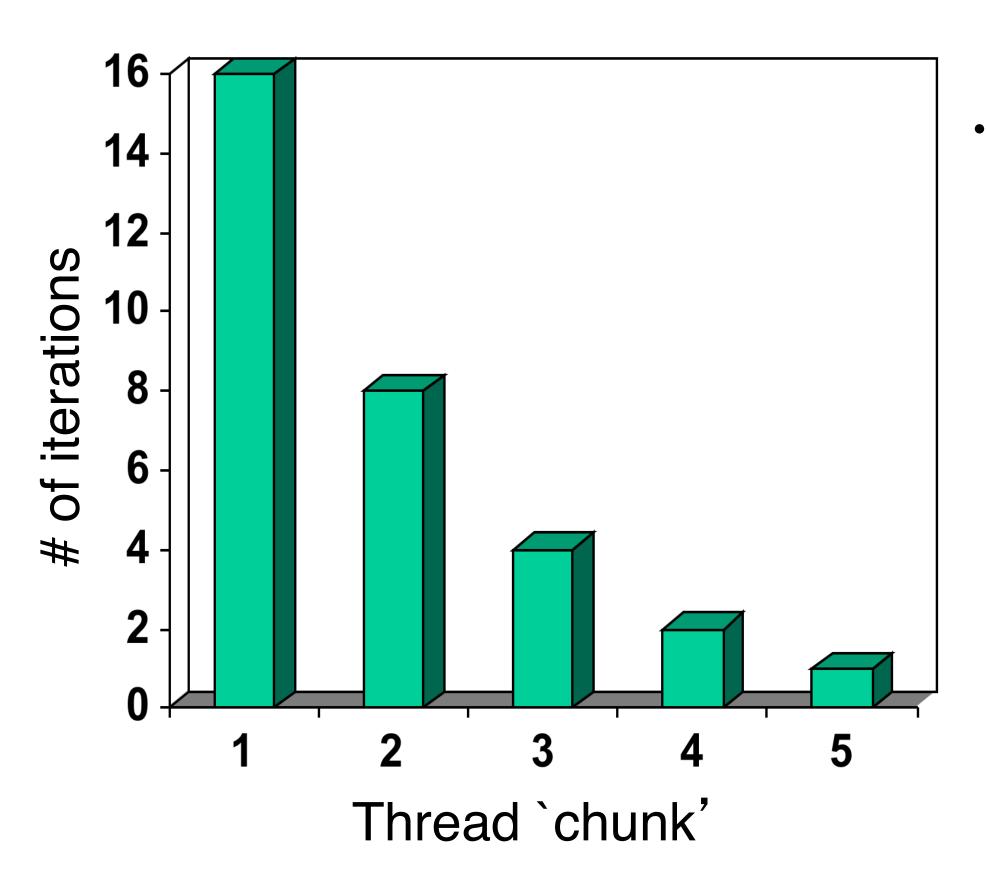
```
#include <omp.h>
static long num steps = 1000000; double step;
#define NUM THREADS 8
void main (){
  int i; double x, pi, sum = 0.0;
  step = 1.0/(double) num_steps;
  omp_set_num_threads(NUM_THREADS);
#pragma omp parallel for private(x) reduction(+:sum)
  for (i=0;i< num steps; i++){
   x = (i+0.5)*step;
    sum = sum + 4.0/(1.0+x*x);
 pi = step * sum;
```

- Notice that we haven't changed any lines of code, only added 4 lines
- Compare to MPI

Dynamic Tasking with OpenMP

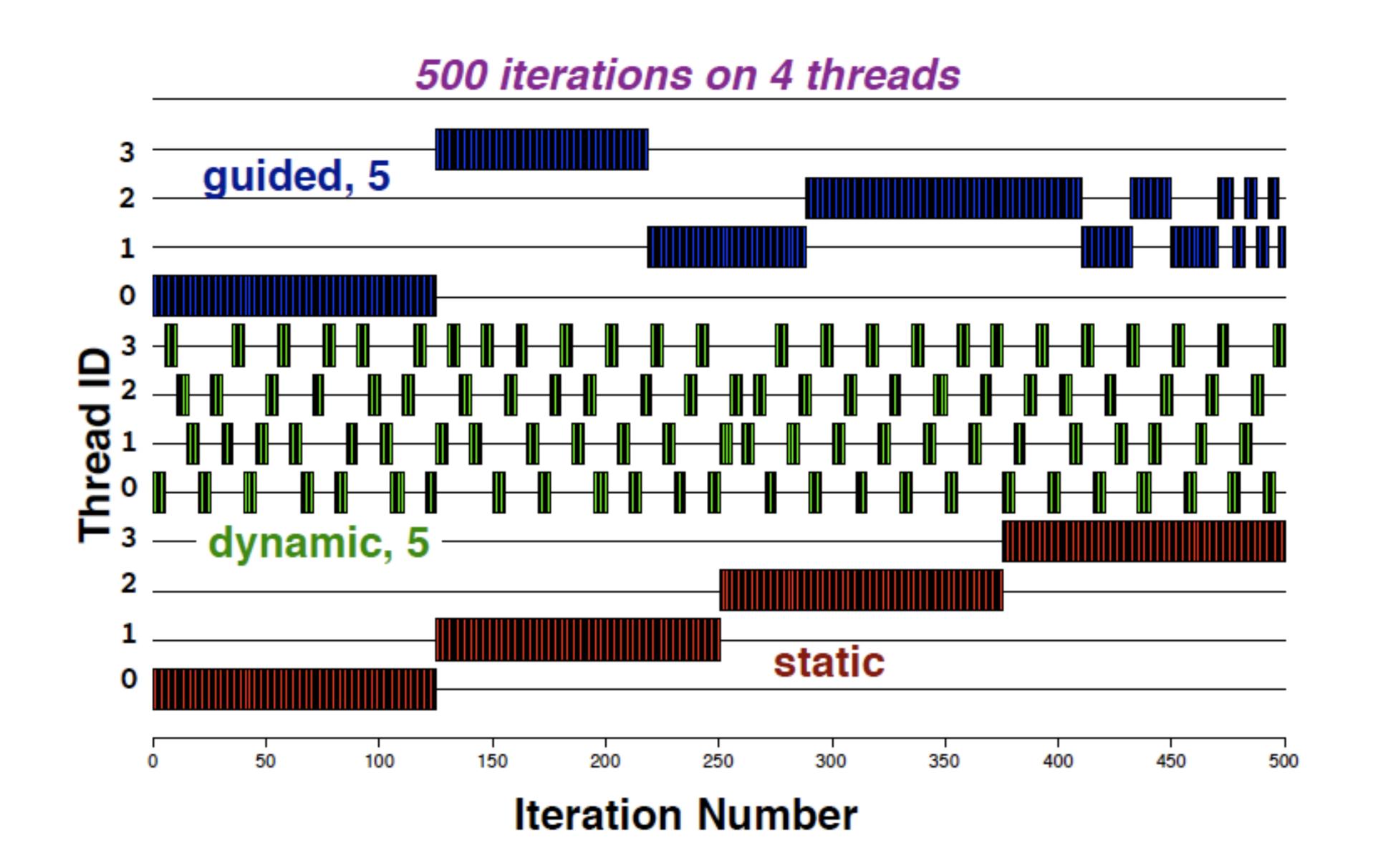
- OpenMP is a mixed bag
 - -schedule(dynamic, size) is a dynamic equivalent to the static directive
 - Master passes off values of iterations to the workers of size size
 - Automatically handles dynamic tasking of simple loops
 - Otherwise must make your own
 - Includes many commonly used cases, unlike static

OpenMP Guided Scheduling



- · schedule(guided, size)
- Guided scheduling is a compromise to reduce scheduling overhead
 - Iteration space is divided up into exponentially decreasing chunks
- Final size is usually 1, unless set by the programmer
 - Chunks of work are dynamically obtained
- Works quite well provided work per iteration is constant - if unknown dynamic is better

OpenMP Scheduling



Tasking in OpenMP 3.0

- Tasking allows parallelization of units of work that are dynamically generated
- Provides flexible model for irregular parallelism
- #pragma omp task [clause [[,]clause] ...] structured-block
- Task Synchronization
 - C/C++: #pragma omp taskwait
 - Current task suspends execution until all children tasks, generated within the current task up to this point, are complete

Fibonacci Example

- Default for local variables is firstprivate

```
int fib (int n)
  int x,y;
  if (n < 2) return n;
#pragma omp task shared(x)
  x = fib(n-1);
#pragma omp task shared(y)
  y = fib(n-2);
#pragma omp taskwait
  return x+y;;
```

OpenMP Summary

- OpenMP provides a simple programming model
 - Loops or sections
 - Incremental parallelism
- Profs. Aiken/Olukotun Targeted at shared memory systems
 - Won't scale easily to large machines
 - Easy to create false sharing
 - Compilers with OpenMP 2.5 support are widely available
 - OpenMP 3.0 supports tasking
 - Supports irregular parallelism

Implementing Barriers

Implementing a centralized barrier

(Barrier for P processors, based on shared counter)

```
struct Barrier_t {
 LOCK lock;
  int counter;
                   // initialize to 0
 int flag;
};
// parameter p gives number of processors that should hit the barrier
void Barrier(Barrier_t* b, int p) {
  lock(b->lock);
  if (b->counter == 0) {
   b->flag = 0; // first thread arriving at barrier clears flag
  int num_arrived = ++(b->counter);
  unlock(b->lock);
  if (num_arrived == p) { // last arriver sets flag
                                                           Does it work? Consider:
   b->counter = 0;
                                                           do stuff ...
   b\rightarrow flag = 1;
                                                           Barrier(b, P);
  else {
                                                           do more stuff ...
   while (b->flag == 0); // wait for flag
                                                           Barrier(b, P);
```

Correct centralized barrier

```
struct Barrier_t {
 LOCK lock;
  int arrive_counter; // initialize to 0 (number of threads that have arrived)
                       // initialize to P (number of threads that have left barrier)
 int leave_counter;
 int flag;
};
void Barrier(Barrier_t* b, int p) {
 lock(b->lock);
  if (b->arrive_counter == 0) {      // if first to arrive...
   if (b->leave_counter == P) {      // check to make sure no other threads "still in barrier"
      b \rightarrow flag = 0;
                                    // first arriving thread clears flag
    } else {
     unlock(lock);
     while (b->leave_counter != P); // wait for all threads to leave before clearing
     lock(lock);
     b\rightarrow flag = 0;
                                     // first arriving thread clears flag
  int num_arrived = ++(b->arrive_counter);
  unlock(b->lock);
  if (num_arrived == p) {
                            // last arriver sets flag
    b->arrive_counter = 0;
    b->leave_counter = 1;
                                                           Main idea: wait for all processes to
   b\rightarrow flag = 1;
                                                           leave first barrier, before clearing
  else {
   while (b->flag == 0); // wait for flag
   lock(b->lock);
                                                          flag for entry into the second
    b->leave_counter++;
   unlock(b->lock);
```

Centralized barrier with sense reversal

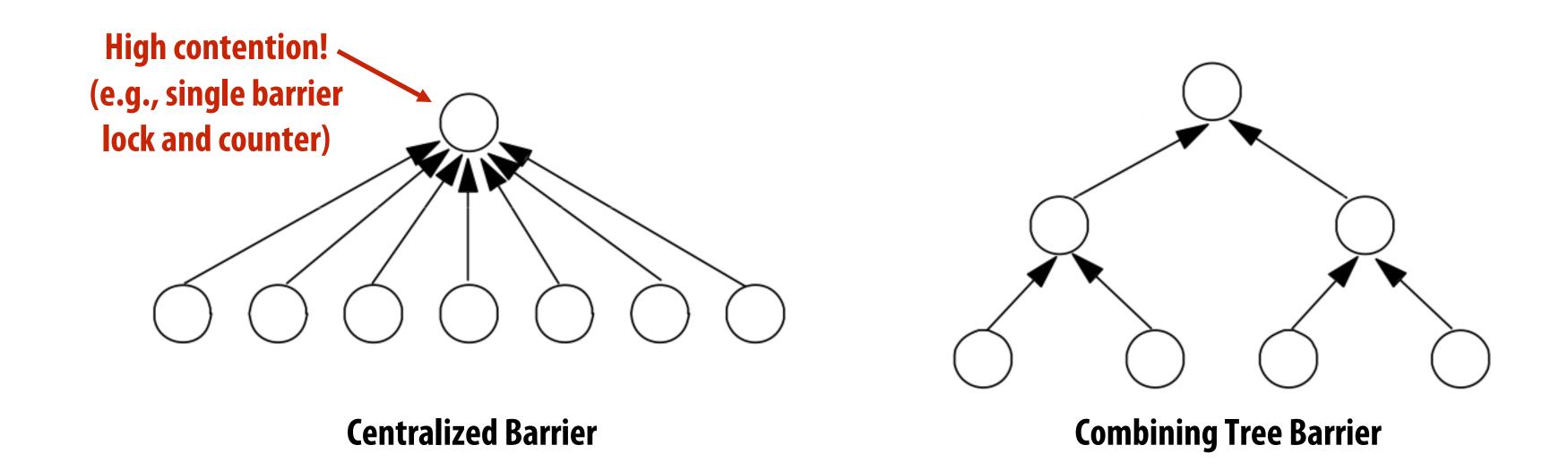
```
struct Barrier_t {
 LOCK lock;
                          // initialize to 0
 int counter;
                            // initialize to 0
 int flag;
int private_sense = 0;  // private per processor. Main idea: processors wait
                             // for flag to be equal to private_sense
void Barrier(Barrier_t* b, int p) {
 private_sense = (private_sense == 0) ? 1 : 0;
  lock(b->lock);
 int num_arrived = ++(b->counter);
  if (b->counter == p) {     // last arriver sets flag
   unlock(b->lock);
   b->counter = 0;
   b->flag = private_sense;
  else {
   unlock(b->lock);
   while (b.flag != private_sense); // wait for flag
```

Sense reversal optimization results in one spin instead of two

Centralized barrier: traffic

- O(P) traffic on interconnect per barrier:
 - All threads: 2P write transactions to obtain barrier lock and update counter
- (O(P) traffic assuming lock acquisition is implemented in O(1) manner)
 - Last thread: 2 write transactions to write to the flag and reset the counter
 (O(P) traffic since there are many sharers of the flag)
 - P-1 transactions to read updated flag
- But there is still serialization on a single shared lock
 - So span (latency) of entire operation is O(P)
 - Can we do better?

Combining tree implementation of barrier



- Combining trees make better use of parallelism in more complex interconnect topologies
 - Ig(P) span (latency)
- Barrier acquire: when processor arrives at barrier, performs increment of parent counter
 - Process recurses to root
- Barrier release: beginning from root, notify children of release