Lecture 10:

Snooping-Based Cache Coherence

Parallel Computing
Stanford CS149, Fall 2019
Intel Core i7

Intel® Core™ i7-3960X Processor Die Detail

- 30% of the die area is cache
Cache hierarchy of Intel Skylake CPU (2015)

64 byte cache line size

Caches exploit locality

L3: (per chip)
- 8 MB, inclusive
- 16-way set associative
- 32B / clock per bank
- 42 cycle latency

L2: (private per core)
- 256 KB
- 4-way set associative, write back
- 64B / clock, 12 cycle latency

L1: (private per core)
- 32 KB
- 8-way set associative, write back
- 2 x 32B load + 1 x 32B store per clock
- 4 cycle latency

Support for:
- 72 outstanding loads
- 56 outstanding stores

Source: Intel 64 and IA-32 Architectures Optimization Reference Manual (June 2016)
Cache design review

Let’s say your code executes `int x = 1;`  
(Assume for simplicity x corresponds to the address 0x12345604 in memory... it’s not stored in a register)

One cache line:

- Do you know the difference between a write back and a write-through cache?
- What about a write-allocate vs. write-no-allocate cache?
Behavior of write-allocate, write-back cache on a write miss (uniprocessor case)

Example: processor executes `int x = 1;`

1. Processor performs write to address that "misses" in cache
2. Cache selects location to place line in cache, if there is a dirty line currently in this location, the dirty line is written out to memory
3. Cache loads line from memory ("allocates line in cache")
4. Whole cache line is fetched and 32 bits are updated
5. Cache line is marked as dirty
Review: Shared address space model (abstraction)

- Threads Reading/writing to shared variables
  - Inter-thread communication is implicit in memory operations
  - Thread 1 stores to X
    - Later, thread 2 reads X (and observes update of value by thread 1)
  - Manipulating synchronization primitives
    - e.g., ensuring mutual exclusion via use of locks

- This is a natural extension of sequential programming
A shared memory multi-processor

- Processors read and write to shared variables
  - More precisely: processors issue load and store instructions

- A reasonable expectation of memory is:
  - Reading a value at address X should return the last value written to address X by any processor
The cache coherence problem

Modern processors replicate contents of memory in local caches

Problem: processors can observe different values for the same memory location

![Diagram of interconnect and caches]

The chart at right shows the value of variable `foo` (stored at address `X`) in main memory and in each processor's cache

Assume the initial value stored at address `X` is 0

Assume write-back cache behavior

<table>
<thead>
<tr>
<th>Action</th>
<th>P1 $</th>
<th>P2 $</th>
<th>P3 $</th>
<th>P4 $</th>
<th>mem[X]</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1 load X</td>
<td>0</td>
<td>miss</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>P2 load X</td>
<td>0</td>
<td>miss</td>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>P1 store X</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>P3 load X</td>
<td>1</td>
<td>0</td>
<td>miss</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>P3 store X</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>P2 load X</td>
<td>1</td>
<td>hit</td>
<td>2</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>P1 load Y</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(assume this load causes eviction of X)
The cache coherence problem

Modern processors replicate contents of memory in local caches

Problem: processors can observe different values for the same memory location

![Diagram showing cache coherence problem]

- Processor 1: Cache
- Processor 2: Cache
- Processor 3: Cache
- Processor 4: Cache
- Interconnect
- Memory

```
int foo;  // (stored at address X)
```

Is this a mutual exclusion problem?

Can you fix the problem by adding locks to your program?

**NO!**

This is a problem created by replicating the data stored at address X in local caches

The chart at right shows the value of variable foo (stored at address X) in main memory and in each processor’s cache

Assume the initial value stored at address X is 0

Assume write-back cache behavior

**How could we fix this problem?**

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<td></td>
<td></td>
<td>0</td>
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<td>0</td>
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<td></td>
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<td>2</td>
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<td>P2 load X</td>
<td>1</td>
<td>0 hit</td>
<td>2</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>P1 load Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

(assume this load causes eviction of X)
The memory coherence problem

- Intuitive behavior for memory system: reading value at address X should return the last value written to address X by any processor.

- Memory coherence problem exists because there is both global storage (main memory) and per-processor local storage (processor caches) implementing the abstraction of a single shared address space.
Intuitive expectation of shared memory

- Intuitive behavior for memory system: reading value at address X should return the last value written to address X by any processor.

- On a uniprocessor, providing this behavior is fairly simple, since writes typically come from one source: the processor
  - Exception: device I/O via direct memory access (DMA)
Coherence is an issue in a single CPU system

Consider I/O device performing DMA data transfer

- Common solutions:
  - CPU writes to shared buffers using uncached stores (e.g., driver code)
  - OS support:
    - Mark virtual memory pages containing shared buffers as not-cachable
    - Explicitly flush pages from cache when I/O completes
- In practice, DMA transfers are infrequent compared to CPU loads and stores
  (so these heavyweight software solutions are acceptable)

Case 1:
Processor writes to buffer in main memory
Processor tells network card to async send buffer
Problem: network card many transfer stale data if processor’s writes (reflected in cached copy of data) are not flushed to memory

Case 2:
Network card receives message
Network card copies message in buffer in main memory using DMA transfer
Card notifies CPU msg was received, buffer ready to read
Problem: CPU may read stale data if addresses updated by network card happen to be in cache
Problems with the intuition

- Intuitive behavior: reading value at address X should return the last value written to address X by any processor.

- What does “last” mean?
  - What if two processors write at the same time?
  - What if a write by P1 is followed by a read from P2 so close in time that it is impossible to communicate the occurrence of the write to P2 in time?

- In a sequential program, “last” is determined by program order (not time)
  - Holds true within one thread of a parallel program
  - But we need to come up with a meaningful way to describe order across threads in a parallel program
Definition: Coherence

A memory system is coherent if:

The results of a parallel program’s execution are such that for each memory location, there is a hypothetical serial order of all program operations (executed by all processors) to the location that is consistent with the results of execution, and:

1. Memory operations issued by any one processor occur in the order issued by the processor

2. The value returned by a read is the value written by the last write to the location... as given by the serial order
Implementation: Cache Coherence Invariants

- **Single-Writer, Multiple-Read (SWMR) Invariant**
  - For any memory location $x$, at any given time (epoch):
    - there exists only a single processor (core) that may write to $x$ (and can also read it)
    - some number of cores that may only read $x$

- **Data-Value Invariant (write serialization)**
  - The value of the memory location at the start of an epoch is the same as the value of the memory location at the end of its last read–write epoch
Implementing coherence

- **Software-based solutions (coarse grain: VM page)**
  - OS uses page-fault mechanism to propagate writes
  - Can be used to implement memory coherence over clusters of workstations
  - We won’t discuss these solutions

- **Hardware-based solutions (fine grain: cache line)**
  - “Snooping”-based coherence implementations (today)
  - Directory-based coherence implementations (next week)
Shared caches: coherence made easy

- One single cache shared by all processors
  - Eliminates problem of replicating state in multiple caches
- Obvious scalability problems (since the point of a cache is to be local and fast)
  - Interference / contention due to many clients (destructive)
- But shared caches can have benefits:
  - Facilitates fine-grained sharing (overlapping working sets)
  - Loads/stores by one processor might pre-fetch lines for another processor (constructive)
SUN Niagara 2 (UltraSPARC T2)

Note area of crossbar (CCX): about same area as one core on chip
Snooping cache-coherence schemes

- Main idea: all coherence-related activity is broadcast to all processors in the system (more specifically: to the processor’s cache controllers)

- Cache controllers monitor ("they snoop") memory operations, and follow cache coherence protocol to maintain memory coherence

Notice: now cache controller must respond to actions from “both ends”:

1. LD/ST requests from its local processor
2. Coherence-related activity broadcast over the chip’s interconnect
Very simple coherence implementation

Let’s assume:

1. **Write-through** caches

2. Granularity of coherence is cache line

Upon write, cache controller broadcasts invalidation message

As a result, the next read from other processors will trigger cache miss

(processor retrieves updated value from memory due to write-through policy)

<table>
<thead>
<tr>
<th>Action</th>
<th>Interconnect activity</th>
<th>P0 $</th>
<th>P1 $</th>
<th>mem location X</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0 load X</td>
<td>cache miss for X</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>P1 load X</td>
<td>cache miss for X</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>P0 write 100 to X</td>
<td>invalidation for X</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>P1 load X</td>
<td>cache miss for X</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

**Write-Invalidate vs. Write-Update**
A clarifying note

The logic we are about to describe is performed by each processor’s cache controller in response to:
- Loads and stores by the local processor
- Messages from other caches

If all cache controllers operate according to this described protocol, then coherence will be maintained
- The caches “cooperate” to ensure coherence is maintained
Cache coherence with write-back caches

- Dirty state of cache line now indicates exclusive ownership
  - Modified: cache is only cache with a valid copy of line (it can safely be written to)
  - Owner: cache is responsible for propagating information to other processors when they attempt to load it from memory (otherwise a load from another processor will get stale data from memory)

What are two important properties of a bus?
Invalidation-based write-back protocol

Key ideas:

- A line in the “modified” state can be modified without notifying the other caches

- Processor can only write to lines in the modified state
  - Need a way to tell other caches that processor wants exclusive access to the line
  - We accomplish this by sending all the other caches messages

- When cache controller sees a request for modified access to a line it contains
  - It must invalidate the line in its cache
Recall cache line state bits

| Line state | Tag | Data (64 bytes on modern Intel processors) |

Dirty bit
MSI write-back invalidation protocol

- **Key tasks of protocol**
  - Ensuring processor obtains exclusive access for a write
  - Locating most recent copy of cache line’s data on cache miss

- **Three cache line states**
  - Invalid (I): same as meaning of invalid in uniprocessor cache
  - Shared (S): line valid in one or more caches, memory is up to date
  - Modified (M): line valid in exactly one cache (a.k.a. “dirty” or “exclusive” state)

- **Two processor operations (triggered by local CPU)**
  - PrRd (read)
  - PrWr (write)

- **Three coherence-related bus transactions (from remote caches)**
  - BusRd: obtain copy of line with no intent to modify
  - BusRdX: obtain copy of line with intent to modify
  - BusWB: write dirty line out to memory
Cache Coherence Protocol: MSI State Diagram

A / B: if action A is observed by cache controller, action B is taken

### Abbreviation | Action
--- | ---
PrRd | Processor Read
PrWr | Processor Write
BusRd | Bus Read
BusRdX | Bus Read Exclusive
BusWB | Bus Writeback
MSI Invalidate Protocol

- Read obtains block in “shared”
  - even if only cached copy

- Obtain exclusive ownership before writing
  - BusRdX causes others to invalidate
  - If M in another cache, will cause writeback
  - BusRdX even if hit in S
    - promote to M (upgrade)

* Remember, all caches are carrying out this logic independently to maintain coherence
## A Cache Coherence Example

<table>
<thead>
<tr>
<th>Proc Action</th>
<th>P1 State</th>
<th>P2 State</th>
<th>P3 State</th>
<th>Bus Act</th>
<th>Data from</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. P1 read x</td>
<td>S</td>
<td>--</td>
<td>--</td>
<td>BusRd</td>
<td>Memory</td>
</tr>
<tr>
<td>2. P3 read x</td>
<td>S</td>
<td>--</td>
<td>S</td>
<td>BusRd</td>
<td>Memory</td>
</tr>
<tr>
<td>3. P3 write x</td>
<td>I</td>
<td>--</td>
<td>M</td>
<td>BusRdX</td>
<td>Memory</td>
</tr>
<tr>
<td>4. P1 read x</td>
<td>S</td>
<td>--</td>
<td>S</td>
<td>BusRd</td>
<td>P3’s cache</td>
</tr>
<tr>
<td>5. P2 read x</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>BusRd</td>
<td>Memory</td>
</tr>
<tr>
<td>6. P2 write x</td>
<td>I</td>
<td>M</td>
<td>I</td>
<td>BusRdX</td>
<td>Memory</td>
</tr>
</tbody>
</table>

- Single writer, multiple reader protocol
- Why do you need Modified to Shared?
- Communication increases memory latency
Summary: MSI

- A line in the M state can be modified without notifying other caches
  - No other caches have the line resident, so other processors cannot read these values
  - (without generating a memory read transaction)

- Processor can only write to lines in the M state
  - If processor performs a write to a line that is not exclusive in cache, cache controller must first broadcast a read-exclusive transaction to move the line into that state
  - Read-exclusive tells other caches about impending write
    (“you can’t read any more, because I’m going to write”)
  - Read-exclusive transaction is required even if line is valid (but not exclusive... it’s in the S state) in processor’s local cache (why?)
  - Dirty state implies exclusive

- When cache controller snoops a “read exclusive” for a line it contains
  - Must invalidate the line in its cache
  - Because if it didn’t, then multiple caches will have the line
    (and so it wouldn’t be exclusive in the other cache!)
Does MSI Satisfy Cache Coherence

1. Single-Writer, Multiple-Read (SWMR) Invariant
   - Only one cache can be in M-state all others get invalidation message
   - Multiple caches can be in read-only S-state

2. Data-Value Invariant (write serialization)
   - On BusRd and BusRdx data is provided by cache with line in M-state
   - Bus serializes all transactions
MESI invalidation protocol

- **MSI** requires two interconnect transactions for the common case of reading an address, then writing to it
  - Transaction 1: BusRd to move from I to S state
  - Transaction 2: BusRdX to move from S to M state

- This inefficiency exists even if application has no sharing at all

**Solution:** add additional state E (“exclusive clean”)
  - Line has not been modified, but only this cache has a copy of the line
  - Decouples exclusivity from line ownership (line not dirty, so copy in memory is valid copy of data)
  - Upgrade from E to M does not require an bus transaction
MESI state transition diagram

- **M (Modified)**
  - PrRd / --
  - PrWr / --

- **E (Exclusive)**
  - PrWr / BusRdX
  - PrWr / --

- **S (Shared)**
  - PrRd / BusRd (no other cache asserts shared)
  - PrRd / --

- **I (Invalid)**
  - PrRd / BusRd (another cache asserts shared)

- **E to I (Invalid)**
  - PrWr / BusRdX
  - PrWr / --

- **S to M (Modified)**
  - BusRd / BusWB

- **M to S (Shared)**
  - BusRd / --

- **E to S (Shared)**
  - BusRdX / --
Two Hard Things

There are only two hard things in Computer Science: cache invalidation and naming things.

-- Phil Karlton
Implications of cache coherence to the programmer
Communication Overhead

- Communication time is key parallel overhead
  - Appears as increased memory latency in multiprocessor
    - Extra main memory accesses in UMA systems
      - Must determine lowering of cache miss rate vs. uniprocessor
    - Some accesses have higher latency in NUMA systems
      - Only a fraction of a % of these can be significant!

![Diagram showing communication overhead between uniprocessor and multiprocessor systems.](image-url)
Unintended communication via false sharing

What is the potential performance problem with this code?

// allocate per-thread variable for local per-thread accumulation
int myPerThreadCounter[NUM_THREADS];

Why might this code be more performant?

// allocate per thread variable for local accumulation
struct PerThreadState {
    int myPerThreadCounter;
    char padding[CACHE_LINE_SIZE - sizeof(int)];
};
PerThreadState myPerThreadCounter[NUM_THREADS];
Demo: false sharing

```c
void* worker(void* arg) {
    volatile int* counter = (int*)arg;
    for (int i=0; i<MANY_ITERATIONS; i++)
        (*counter)++;
    return NULL;
}

void test1(int num_threads) {
    pthread_t threads[MAX_THREADS];
    int counter[MAX_THREADS];
    for (int i=0; i<num_threads; i++)
        pthread_create(&threads[i], NULL, &worker, &counter[i]);
    for (int i=0; i<num_threads; i++)
        pthread_join(threads[i], NULL);
}

void test2(int num_threads) {
    pthread_t threads[MAX_THREADS];
    padded_t counter[MAX_THREADS];
    for (int i=0; i<num_threads; i++)
        pthread_create(&threads[i], NULL, &worker, &(counter[i].counter));
    for (int i=0; i<num_threads; i++)
        pthread_join(threads[i], NULL);
}
```

Execution time with num_threads=8 on 4-core system: 14.2 sec
Execution time with num_threads=8 on 4-core system: 4.7 sec
False sharing

- Condition where two processors write to different addresses, but addresses map to the same cache line
- Cache line “ping-pongs” between caches of writing processors, generating significant amounts of communication due to the coherence protocol
- No inherent communication, this is entirely *artifactual* communication (cachelines > 4B)
- False sharing can be a factor in when programming for cache-coherent architectures
Summary: snooping-based coherence

- The cache coherence problem exists because the abstraction of a single shared address space is not implemented by a single storage unit
  - Storage is distributed among main memory and local processor caches
  - Data is replicated in local caches for performance

- Main idea of snooping-based cache coherence: whenever a cache operation occurs that could affect coherence, the cache controller broadcasts a notification to all other cache controllers in the system
  - Challenge for HW architects: minimizing overhead of coherence implementation
  - Challenge for SW developers: be wary of artifactual communication due to coherence protocol (e.g., false sharing)

- Scalability of snooping implementations is limited by ability to broadcast coherence messages to all caches!
  - In a future lecture: scaling cache coherence via directory-based approaches
What is OpenMP?

- OpenMP is a pragma based API that provides a simple extension to C/C++ and FORTRAN
- It is designed for shared memory programming focusing on loops
- OpenMP is a very simple interface to threads based programming
  - Compiler directives
  - Environment variables
  - Run time routines
Loop Parallelism (LLP)

- Overwhelming majority of scientific/engineering applications are expressed in terms of iterative constructs, that is, loops
  - Focus on parallelizing loops

- Particular useful approach if starting from an existing program
  - Major restructuring is impractical/unnecessary

- Goal of exploiting LLP is to evolve the sequential program into a parallel program
  - Through transformations that leave the program semantics unchanged

- LLP works well for shared address space (e.g. Multicore)
General Approach for Loop Parallelism

1. Find the hotspots
2. Eliminate loop-carried dependencies
3. Parallelize the loops
4. Optimize the loop schedule
Find the Hotspots

- By code inspection
- By using performance analysis tools
Parallel Loops

- for (i = 0; i < n; i++) {
}

- for (i = 1; i < n; i++) {
  A[i] = A[i-1] + C[i-1]; /* S1 */
  B[i] = B[i-1] + A[i]; /* S2 */
}
Parallel Loops

- for (i = 0; i < n; i++) {
  A[i] = A[i] + B[i]; /* S1 */
  B[i+1] = C[i] + D[i]; /* S2 */
}

B[n] = C[n-1] + D[n-1];

- for (i = 0; i < n-1; i++) {
  B[i+1] = C[i] + D[i]; /* S2 */
  A[i+1] = A[i+1] + B[i+1]; /* S1 */
}

B[n] = C[n-1] + D[n-1];
For-loop with independent iterations

```c
for (i = 0; i < n; i++)
    c[i] = a[i] + b[i];
```

For-loop parallelized using an OpenMP pragma

```c
#pragma omp parallel for
    shared(n, a, b, c)
    private(i)
for (i = 0; i < n; i++)
    c[i] = a[i] + b[i];
```

```
% cc -xopenmp source.c
% setenv OMP_NUM_THREADS 4
% a.out

gcc source.c -fopenmp
```
Privatizing Variables

- Critical to performance!
- OpenMP pragmas:
  - Designed to make parallelizing sequential code easier
  - Makes copies of “private” variables *automatically*
    - And performs some automatic initialization, too
  - Must specify shared/private per-variable in parallel region
    - private: Uninitialized private data
      - Private variables are undefined on entry and exit of the parallel region
    - shared: All-shared data
    - threadprivate: “static” private for use across several parallel regions
Firstprivate/Lastprivate Clauses

- **firstprivate** (list)
  - All variables in the list are initialized with the value the original object had before entering the parallel region

- **lastprivate** (list)
  - The thread that executes the last iteration or section in sequential order updates the value of the objects in the list
main()
{
    A = 10;

#pragma omp parallel
{
    #pragma omp for private(i) firstprivate(A) lastprivate(B)...
    for (i=0; i<n; i++)
    {
        ....
        B = A + i; /*-- A undefined, unless declared firstprivate --*/
        ....
    }
    C = B;     /*-- B undefined, unless declared lastprivate --*/
}
} /*-- End of OpenMP parallel region --*/
for directive Example

```cpp
#pragma omp parallel default(none)
 shared(n,a,b,c,d) private(i)
{
    #pragma omp for
    for (i=0; i<n-1; i++)
        b[i] = (a[i] + a[i+1])/2;

    #pragma omp for
    for (i=0; i<n; i++)
        d[i] = 1.0/c[i];

} /* End of parallel region */

(Implied barrier)
Nested Loop Parallelism

```c
#pragma omp parallel for
for(int y=0; y<25; ++y)
{
    #pragma omp parallel for
    for(int x=0; x<80; ++x)
        tick(x,y);
}
```
Multiple Part Parallel Regions

- You can also have a “multi-part” parallel region
  - Allows easy alternation of serial & parallel parts
  - Doesn’t require re-specifying # of threads, etc.

```c
#pragma omp parallel . . .
{
  #pragma omp for
  . . . Loop here . . .
  #pragma omp single
  . . . Serial portion here . . .
  #pragma omp sections
  . . . Sections here . . .
}
```
OMP Directives Overheads

![Graph showing overheads for different OMP directives with respect to the number of threads.]
“if” Clause

- if (scalar expression)
  - Only execute in parallel if expression evaluates to true
  - Otherwise, execute serially

```c
#pragma omp parallel if (n > threshold) \
    shared(n, x, y) private(i)
{
    #pragma omp for
    for (i=0; i<n; i++)
        x[i] += y[i];
} /*-- End of parallel region --*/
```
Reductions in OpenMP

- May add reduction clause to parallel for pragma
- Specify reduction operation and reduction variable
- OpenMP takes care of storing partial results in private variables and combining partial results after the loop
- The reduction clause has this syntax:
  reduction (<op> :<variable>)

- Operators
  - + Sum
  - * Product
  - &, |, ^ Bitwise and, or, exclusive or
  - &&, || Logical and, or
Example: Numerical Integration

- We know mathematically that

\[ \pi = \int_0^1 \frac{4.0}{1 + x^2} \, dx \]

\[ \sum_{i=0}^{N} F(x_i) \Delta x \approx \pi \]

- We can approximate the integral as a sum of rectangles:
static long num_steps = 100000;
double step;

void main () {
   int i; double x, pi, sum = 0.0;
   step = 1.0/(double) num_steps;
   for (i=0; i< num_steps; i++){
      x = (i+0.5)*step;
      sum = sum + 4.0/(1.0+x*x);
   }
   pi = step * sum;
}
Loop Parallelized Pi Computation

- Notice that we haven’t changed any lines of code, only added 4 lines
- Compare to MPI

```c
#include <omp.h>
static long num_steps = 1000000; double step;
#define NUM_THREADS 8

void main (){  
    int i; double x, pi, sum = 0.0;
    step = 1.0/(double) num_steps;
    omp_set_num_threads(NUM_THREADS);
    #pragma omp parallel for private(x) reduction(+:sum)
    for (i=0;i< num_steps; i++){
        x = (i+0.5)*step;
        sum = sum + 4.0/(1.0+x*x);
    }
    pi = step * sum;
}  
```